Phys 23.02 quick lecture

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2025 W34–[1]

[1]Phys 23.02. All figures are from Young and Freedman (2019) unless noted.

Agenda

Units, physical quantities, and vectors [E1] 🌀

Motion along a straight line [E2] 🚃

Motion in two or three dimension [E4] 📈

Newton's laws of motion

Applying Newton's laws [E3] 🐌

Work and kinetic energy

Potential energy and energy conservation

Momentum, impulse, and collisions [E5] 💥

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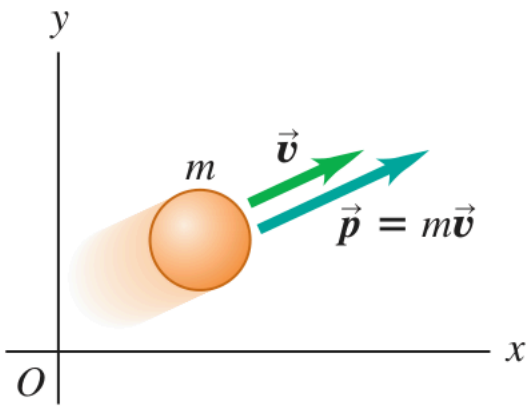
Momentum of a particle

- The momentum $\vec{p}$ of a particle is a vector quantity equal to the product of the particle's mass m and velocity $\vec{v}$

$$\vec{p} = m\vec{v}$$



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

Momentum of a particle

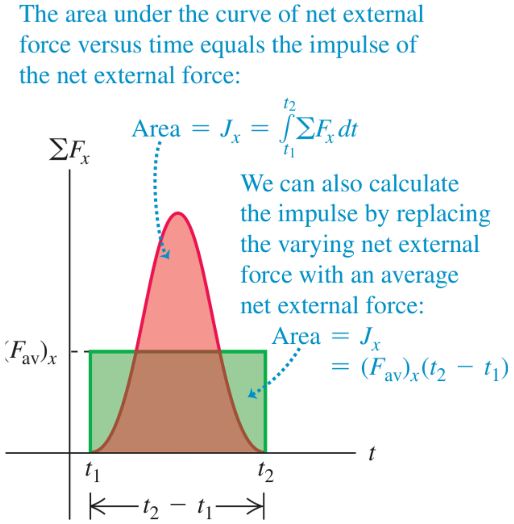
- Newton's second law says that the net external force on a particle is equal to the rate of change of the particle's momentum

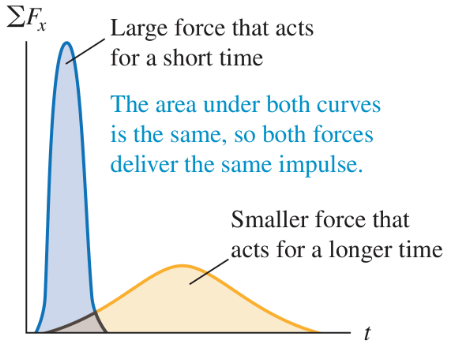
$$\sum \vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

Impulse and momentum

- If a constant net external force $\sum \vec{F}$ acts on a particle for a time interval Δt from t_1 to t_2, the impulse $\vec{J}$ of the net external force is the product of the net external force and the time interval

$$\vec{J} = \sum \vec{F}(t_2 - t_1) = \sum \vec{F} \Delta t$$





Impulse and momentum

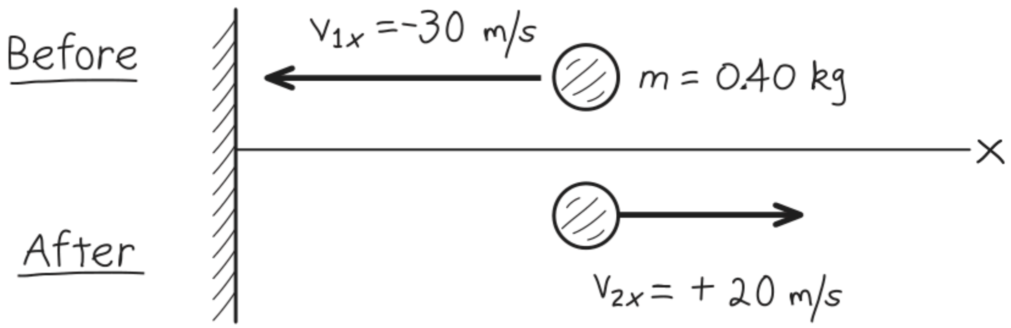
- If $\sum \vec{F}$ varies with time, $\vec{J}$ is the integral of the net external force over the time interval

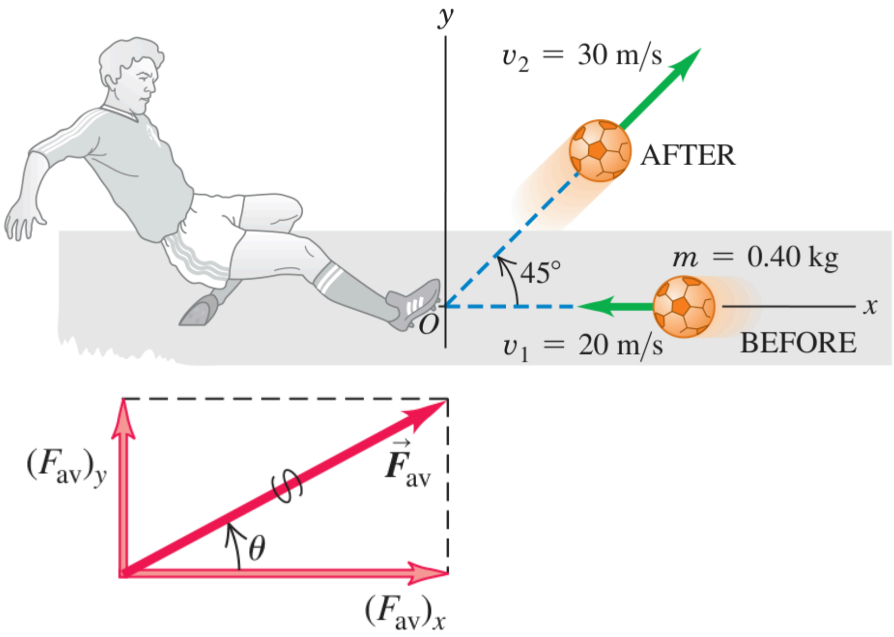
$$\vec{J} = \int_{t_1}^{t_2} \sum \vec{F} \, \mathrm{d}t$$

- In any case, the change in a particle's momentum during a time interval equals the impulse of the net external force that acted on the particle during that interval

$$\vec{J} = \vec{p}_2 - \vec{p}_1$$

- The momentum of a particle equals the impulse that accelerated it from rest to its present speed
- See Examples 8.1–8.3

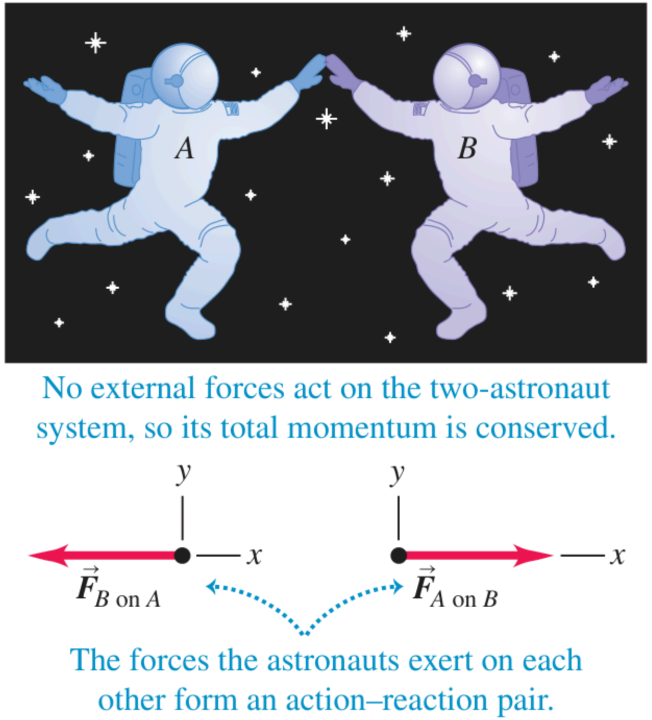




Conservation of momentum

- An internal force is a force exerted by one part of a system on another
- An external force is a force exerted on any part of sys by something outside the system

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots$$
$$= m_A\vec{v}_A + m_B\vec{v}_B + \cdots$$

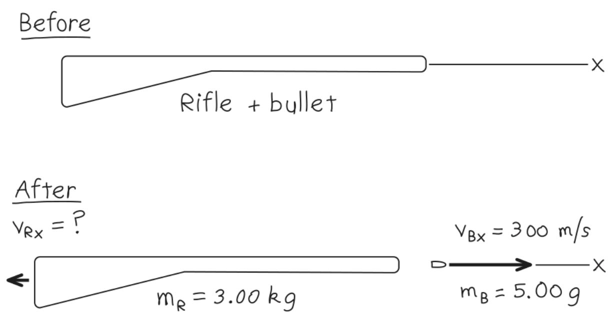


Conservation of momentum

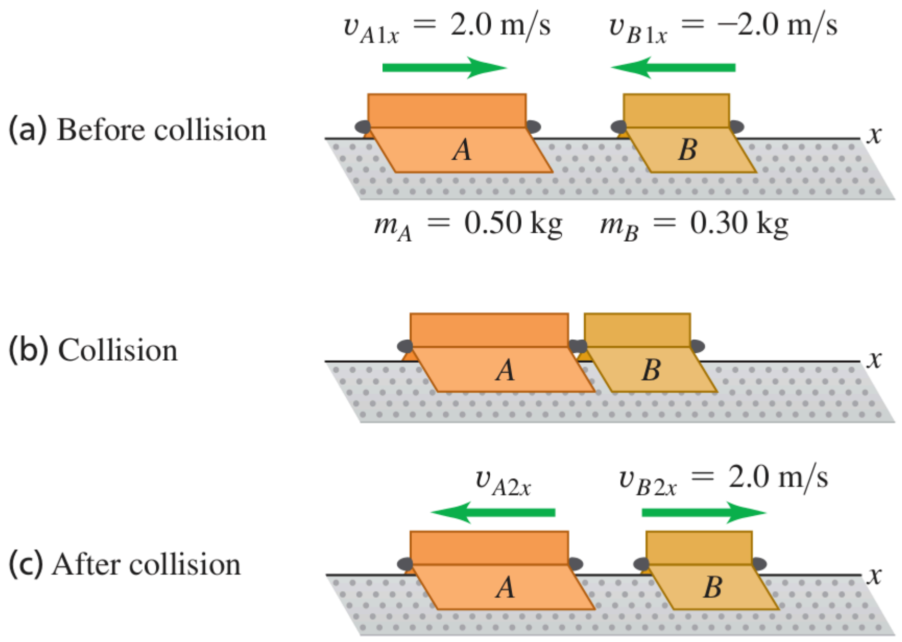
- If the net external force on a system is zero, the total momentum of the system $\vec{P}$ (the vector sum of the momenta of the individual particles that make up the system) is constant, or conserved

$$\sum \vec{F} = \vec{0} \implies \vec{P} = \text{constant}$$

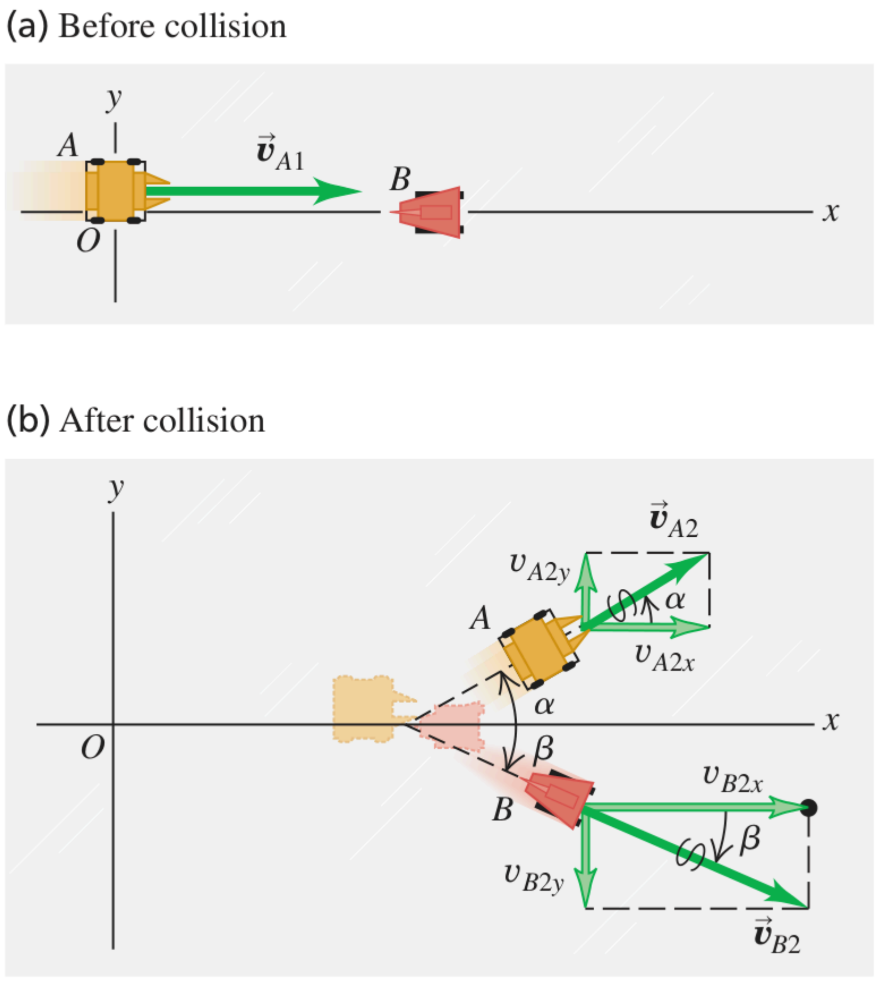
- Each component of total momentum is separately conserved
- See Examples 8.4–8.6



Before
Rifle + bullet
x
After
$v_{Rx} = ?$
$v_{Bx} = 300$ m/s
$m_R = 3.00$ kg
$m_B = 5.00$ g
x



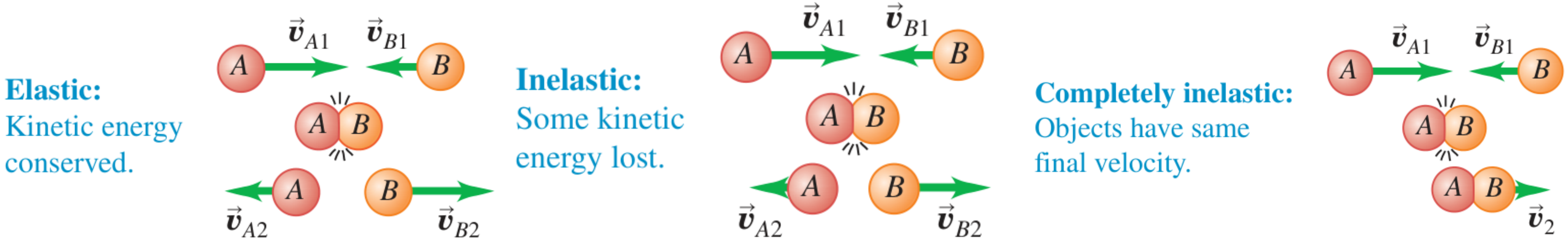
$v_{A1x} = 2.0$ m/s
$v_{B1x} = -2.0$ m/s
(a) Before collision
A
B
x
$m_A = 0.50$ kg
$m_B = 0.30$ kg
(b) Collision
A
B
x
v_{A2x}
$v_{B2x} = 2.0$ m/s
(c) After collision
A
B
x



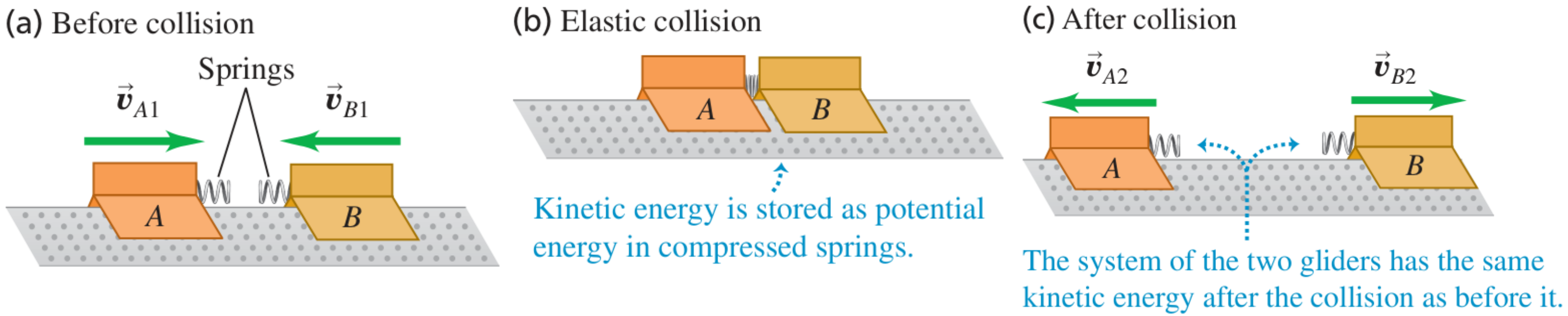
(a) Before collision
y
A
$\vec{v}_{A1}$
B
O
x
(b) After collision
y
$\vec{v}_{A2}$
v_{A2y}
α
A
v_{A2x}
α
x
O
β
v_{B2x}
B
β
v_{B2y}
$\vec{v}_{B2}$

Collisions

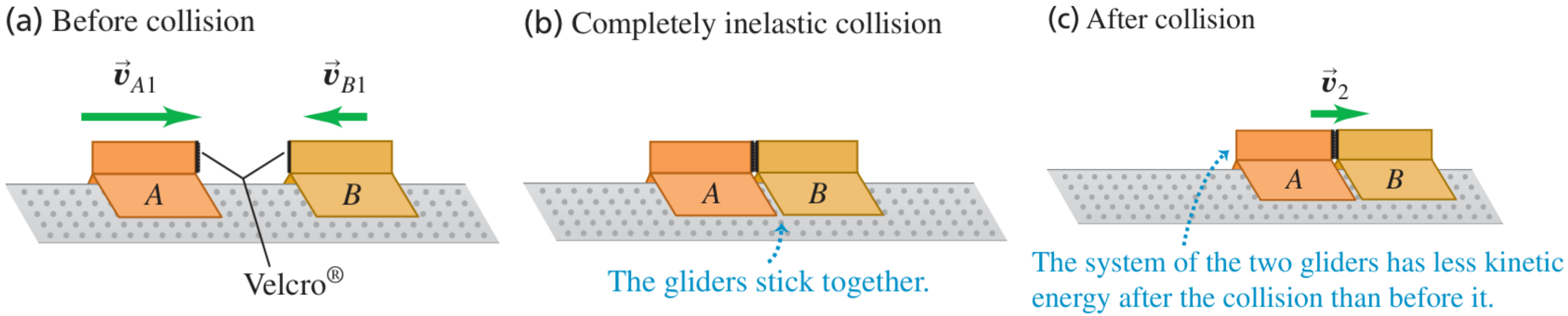
- In typical collisions, initial and final total momenta are equal



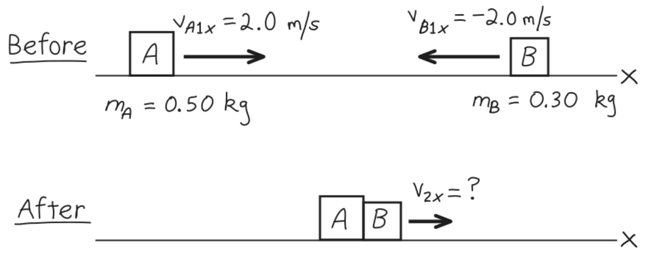
- In an elastic collision between two objects, the initial and final total kinetic energies are also equal, and the initial and final relative velocities have the same magnitude

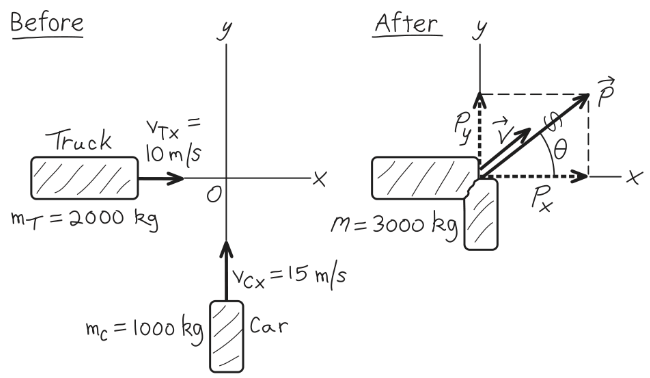


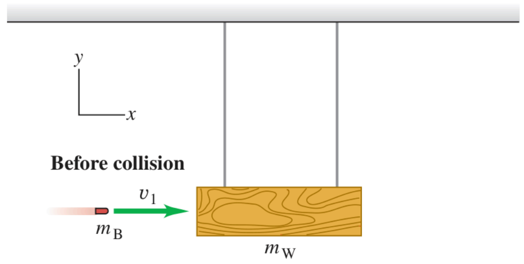
- In an inelastic two-object collision, the total kinetic energy is less after the collision than before

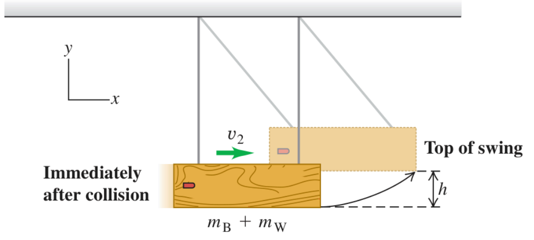


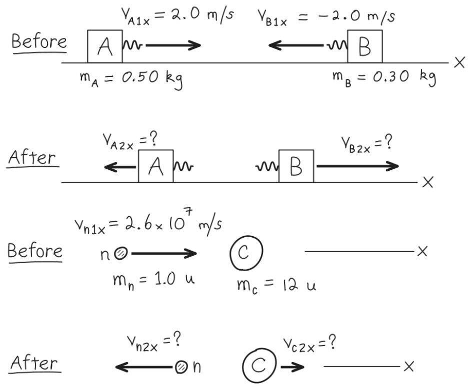
- If the two objects have the same final velocity, the collision is completely inelastic
- See Examples 8.7–8.12

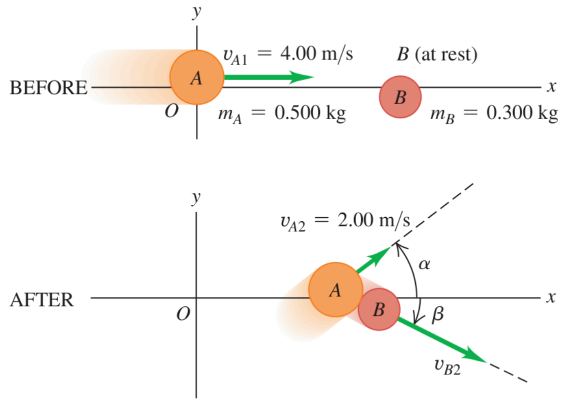












Center of mass

- The position vector of the center of mass of a system of particles $\vec{r}_{\text{cm}}$ is a weighted average of the positions $\vec{r}_1, \vec{r}_2, \dots$ of the individual particles

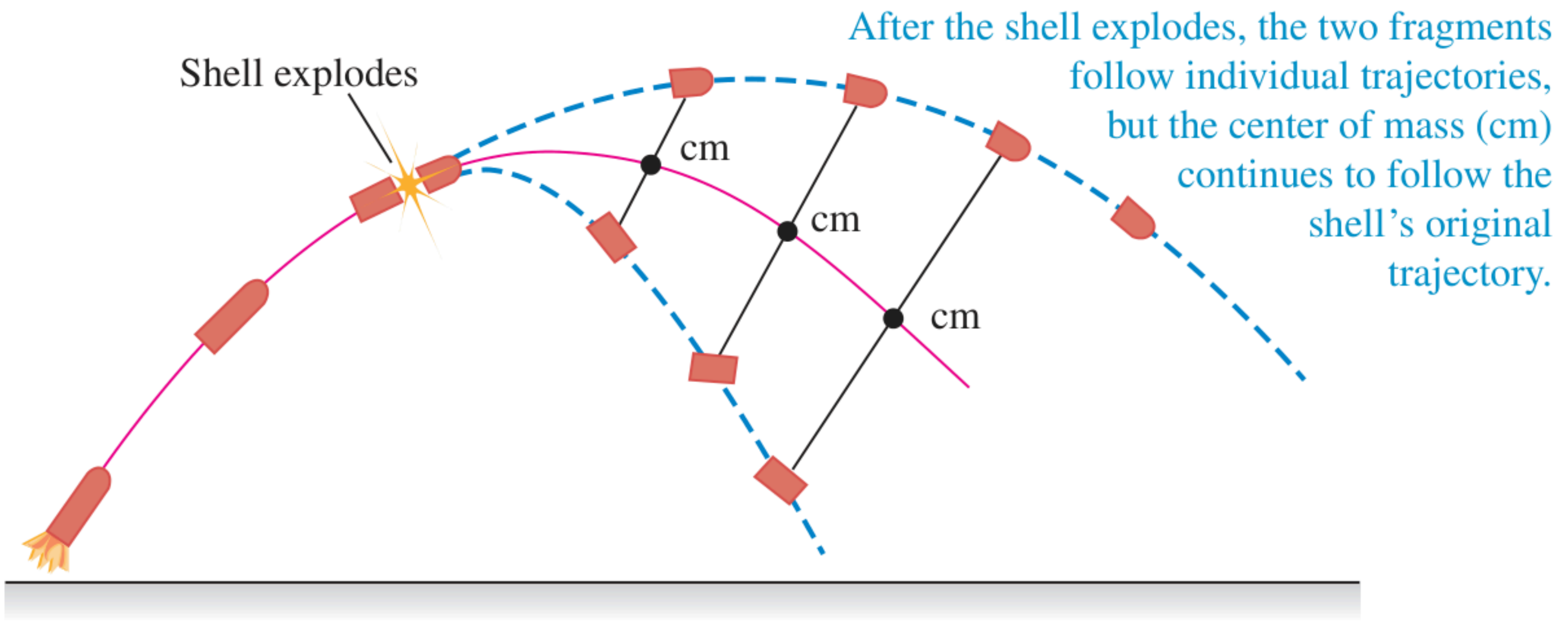
$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

- The total momentum $\vec{P}$ of a system equals the system's total mass M multiplied by the velocity of its center of mass $\vec{v}_{\text{cm}}$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_3 + m_3\vec{v}_3 + \cdots = M\vec{v}_{\text{cm}}$$

- The center of mass moves as though all the mass M were concentrated at that point
- If the net external force on the system is zero, the center-of-mass velocity $\vec{v}_{\text{cm}}$ is constant
- If the net external force is not zero, the center of mass accelerates as though it were a particle of mass M being acted on by the same net external force

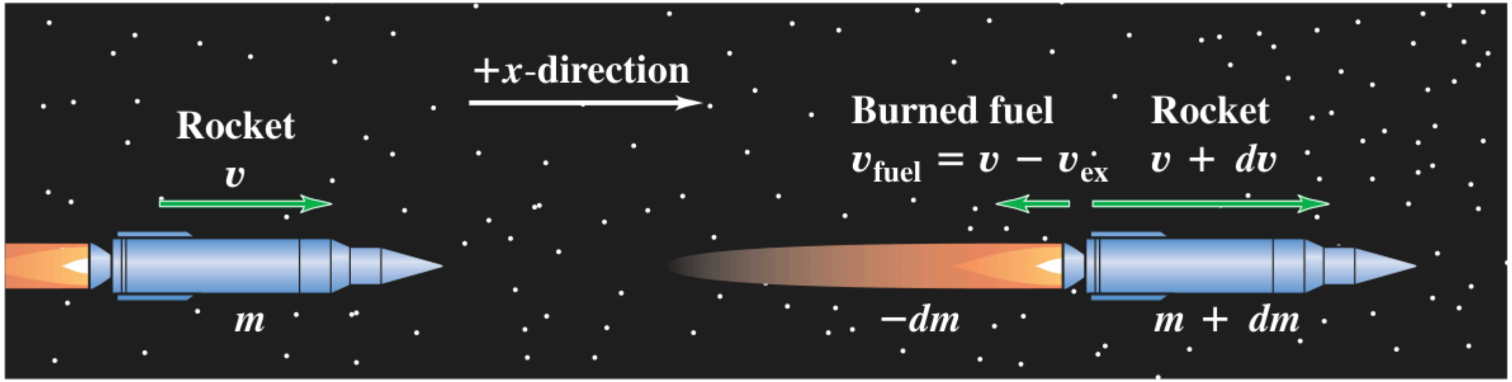
$$\sum \vec{F}_{\text{ext}} = M\vec{a}_{\text{cm}}$$



- See Examples 8.13 and 8.14

Rocket propulsion

- In rocket propulsion, the mass of a rocket changes as the fuel is used up and ejected from the rocket
- Analysis of the motion of the rocket must include the momentum carried away by the spent fuel as well as the momentum of the rocket itself



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{fuel} = v - v_{ex}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

- See Examples 8.15 and 8.16