Phys 20.01 lecture notes

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These lecture notes for Phys 20.01 (Elementary Physics I) offer a deeper dive into the topics than the class slides. They will be updated regularly throughout the course, drawing from

- P.P. Urone and R. Hinrichs, College Physics 2e, OpenStax (2022), open access
- P.G. Hewitt, Conceptual Physics 13e, Pearson (2021)
- H.D. Young and R.A. Freedman, University Physics with Modern Physics 15e, Pearson (2019).

The topics include:

1. Kinematics

This module explores one and two dimensional kinematics, specifically on

- displacement, velocity, acceleration (2.1–2.4)
- motion with constant acceleration, falling objects (2.5, 2.7)
- vectors, motion in two dimensions (3.1–3.3)
- projectile motion, velocity addition (3.4, 3.5)
- Problem solving, graphical analysis (2.6, 2.8) ⋆.

1.1. One-dimensional kinematics

- Motion is everywhere we look
  - Obvious: tennis game, GPS satellites
  - Subtle: blood flowing in your veins despite you resting, molecules vibrating within an inanimate table
- **Kinematics** is the study of motion without considering its causes
  - Usually, causes = forces
  - **Dynamics** is such study with causes
- **Position** is where an object is at any particular time
  - More precisely, where is the object relative to a convenient reference frame
  - Reference frame is a perspective from which you're making observations and is composed of an origin, a set of axes, and a clock. You can imagine said frame is a coordinate system you choose to describe spacetime
  - Often, we take another stationary object in said frame eg. rocket launch relative to earth
- **Displacement** Δx is the change in position of an object. In symbols,

$$\Delta x = x_f - x_0$$

  where x_0 and x_f are the initial and final positions
  - Btw, Δx means change in x, and x_0 is read as "x naught"
  - Object moves relative to a reference frame ⇒ object's position changes ⇒ object has been "displaced" ⇒ displacement
  - Its SI unit is meter (m), but feel free to use km, mi, ft, etc. as long as you convert properly and use such units consistently

- It is a vector quantity ⇒ has magnitude and direction
- Choose which direction is positive, usually it is the rightward or upward direction

- **Distance traveled** is the total length of the path traveled between two positions
  - Distance is the magnitude of displacement between two positions
  - Recall that displacement only considers the initial and final positions
  - Thus, distance traveled between two positions is not necessarily the same as the distance between them (that is the magnitude of displacement). They're equal if path is straight, otherwise former can be greater than the latter
  - Both are scalar quantities ⇒ only magnitude and no direction (and parity $\pm$)
  - In kinematics, we almost always deal with displacement and distance, not distance traveled
- A **vector** is any quantity with both magnitude and direction eg. displacement, velocity, force
  - A **scalar** is any quantity that has magnitude but no direction eg. distance, temperature, energy, speed, height
  - In 1-dim, the direction of a vector is represented simply by parity $\pm$
  - Generally and graphically, it is represented by arrows. The arrow has a length proportional to vector's magnitude and points in the vector's direction
  - Scalars can be negative but parity indicates a point on a scale rather than a direction
- A **coordinate system** must be assigned in a reference frame in order to describe a vector
  - In 1-dim, positive is typically the rightward direction for horizontal motion and upward for vertical motion
  - The opposite direction can be chosen if more convenient, just be consistent throughout
- **Time** is measured in terms of change and is the interval over which change occurs. Elapsed time Δt for an event of change is

  $$\Delta t = t_f - t_0$$

  where t_0 and t_f are times at the beginning and end of event
  - Its SI unit is second (s)
  - For convenience and by convention, initial time t_0 is often taken to be zero ($t_0 = 0$) as if measured with a stopwatch. Elapsed time is just then $\Delta t = t_f \equiv t$
- **Velocity**, specifically average velocity $\overline{v}$, is defined as displacement divided by travel time. In symbols,

  $$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0}$$

  - Its SI unit is m/s, and it is a vector since displacement is a vector
  - It only considers the initial and final positions, providing no info about the motion in between. By considering smaller segments of the motion over shorter time intervals, we obtain more info. This is analogous to the derivative in calculus which considers infinitesimally small interval
  - Instantaneous velocity v is the velocity at a specific instant of time, or is the average velocity for an infinitesimal interval, as in

    $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

  - Instantaneous speed is the magnitude of the instantaneous velocity
  - Average speed is the total distance traveled divided by travel time
  - It is not the magnitude of average velocity since velocity makes use of displacement, not distance traveled

    - Speed, be it average or instantaneous, is a scalar
- **Acceleration**, specifically average acceleration $\overline{a}$ is the rate at which velocity changes. In symbols,

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$$

    - It is a vector, and its SI unit is $\mathrm{m/s^2}$
    - It can be caused by a change in the velocity's magnitude, direction, or both
    - Instantaneous acceleration a is the acceleration at a specific instant of time

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{\mathrm{d}v}{\mathrm{d}t}$$

    - While acceleration is in the direction of the change in velocity, it is not always in the direction of an object's motion
    - Deceleration is an acceleration whose direction is opposite to that of the velocity, resulting in a decrease in velocity. Simply put, it acts in the opposite direction to the object's motion, causing it to slow down
    - Negative acceleration is acceleration in the negative direction in the chosen coordinate system, and is not necessarily deceleration. That is, it does not necessarily cause an object to slow down
- In 1-dim, the **kinematic equations** for motion with **constant** $\boldsymbol{a}$ are

$$x = x_0 + \overline{v}t, \qquad \overline{v} = \frac{1}{2}(v_0 + v), \qquad v = v_0 + at,$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2, \qquad v^2 = v_0^2 + 2a(x - x_0)$$

    - Here, we take acceleration to be constant so $\overline{a} = a$ at all times, and initial time to be zero
    - In vertical motion, y takes the place of x
- An object in **free-fall** experiences constant a if air resistance is negligible
    - Free-fall is the state of motion resulting from only gravitational force
    - On earth, all free-falling objects have an acceleration a_g due to gravity, which averages at $a_g = 9.8\mathrm{m/s^2}$
    - Acceleration a_g can be taken either as $+a_g$ or $-a_g$ depending on your choice of coordinate system. If you choose upward to be positive, $a = -a_g$ is negative, otherwise $a = a_g$. Former is the typical choice
    - Since a is constant in free-fall, you can use above kinematic equations where either $a = \pm a_g$

1.2. Two-dimensional kinematics

- In 2-dim, the shortest path between any two points is a straight line
    - This path can be represented by a vector with **horizontal and vertical components**
    - These components are independent of one another. Relating to kinematics, motion in the horizontal direction does not affect motion in the vertical, and vice versa
- **Vectors** quantities have magnitude and direction, and combine as per rules of vector addition
    - Scalar quantities are just magnitudes and combine as per usual rules of arithmetic
    - The negative of a vector has the same magnitude but points in the opposite direction, eg. deceleration

    - Graphically, adding vectors $\vec{A}$ and $\vec{B}$ results in the resultant vector $\vec{R} = \vec{A} + \vec{B}$ obtained by placing the tail of $\vec{B}$ at the head of $\vec{A}$ and drawing $\vec{R}$ from the tail of $\vec{A}$ to the head of $\vec{B}$, or the other way around
    - Vectors can be added by using components of vectors. The x-component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x-components of $\vec{A}$ and $\vec{B}$, and likewise for y (and even z), as in

$$R_x = A_x + B_x, \qquad R_y = A_y + B_y, \qquad R_z = A_z + B_z$$

- TODO: graphical methods of vector addition
- TODO: analytical methods of vector addition
- An object is in **projectile motion** through the air if it is subject only to gravitational acceleration a_g. To solve problems involving this motion, try to perform the following
    - Determine the coordinate system
    - Analyze the horizontal component using

$$a_x = 0, \qquad x = x_0 + v_x t, \qquad v_x = v_{0x} \text{ (constant velocity)}$$

    - Analyze the vertical component using

$$a_y = -a_g = -9.8 \text{ m/s}^2, \qquad y = y_0 + \frac{1}{2}(v_{0y}t + v_y)t, \qquad v_y = v_{0y} - a_g t,$$
$$y = y_0 + v_{0y}t - \frac{1}{2}a_g t^2, \qquad v_y^2 = v_{0y}^2 - 2a_g(y - y_0)$$

    assuming upward is positive
    - Recombine horizontal and vertical components of position and velocity using

$$s = \sqrt{x^2 + y^2}, \qquad \theta = \arctan\left(\frac{y}{x}\right), \qquad v = \sqrt{v_x^2 + v_y^2}, \qquad \theta_v = \arctan\left(\frac{v_y}{v_x}\right)$$

    - The maximum height $h \equiv \max(y)$ of a projectile launched with initial vertical velocity v_{0y} is

$$h = \frac{v_{0y}^2}{2a_g}$$

    - The maximum horizontal distance traveled by a projectile is called range $R \equiv \max(x)$. The R of a projectile on a level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is

$$R = \frac{v_0^2}{a_g}\sin(2\theta_0)$$

1.3. Problem solving, graphical analysis ⋆

- The problem-solving basics for physics problems are
    - Examine the situation to determine which physical principles are involved
    - Make a list of what is given or can be inferred from the problem as stated (identify the knowns)
    - Identify exactly what needs to be determined in the problem (identify the unknowns)
    - Find an equation or set of equations that can help you solve the problem
    - Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units
    - Check the answer to see if it is reasonable: does it make sense?
- Graphs of motion can be used to analyze motion

    - In fact, graphical solutions correspond to solutions that are obtainable via the mathematical methods and equations we learned
    - Recall that y-intercept is the y-value when $x = 0$ or when a line crosses the y-axis. Also recall that slope m is the difference in y-value (rise) divided by the difference in x-value (run) between two points on a straight line. In symbols,

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$$

    - The slope of a graph of displacement x vs time t is velocity v
    - The slope of a graph of velocity v vs time t is acceleration a
    - Average velocity, instantaneous velocity, and acceleration can be obtained by graphical analysis
- The 3 fundamental **physical quantities** are mass, length, and time
    - Their corresponding SI units are kilogram (kg), meter (m), and second (s)
    - Derived units for other physical quantities are products or quotients of the basic units
    - Equations must be dimensionally consistent, that is two terms can be added only when they have the same units
- TODO: rules for the sign for velocity, acceleration, etc.