Fluid dynamics

R. Torres

2025 W47[1]

[1]Phys 20.01 Mod 5. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously

Fluid flow

Bernoulli's equation

Viscosity and turbulence

Quiz time 🕐

Previously

Fluids, density, pressure, buoyancy

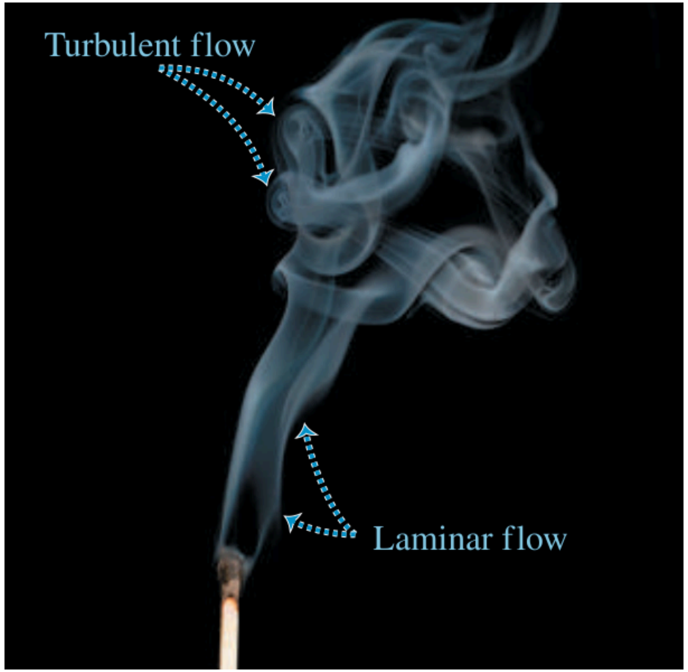
Fluid flow

Moving fluids

- We are now ready to consider *motion* of a fluid
- Many fluids are flowing in this scene 🌊
  - Water from hose & smoke from fire are visible flows
  - Less visible are flow of air, of fluids on ground, and of fluids within firefighters 🩸

Moving fluids

- Fluid flow can be extremely complex. We can represent some situations by simple idealized models
- **Ideal fluid** is a fluid that is *incompressible* & not *viscous*
  - Incompressible means its density cannot change
  - Viscosity is internal friction

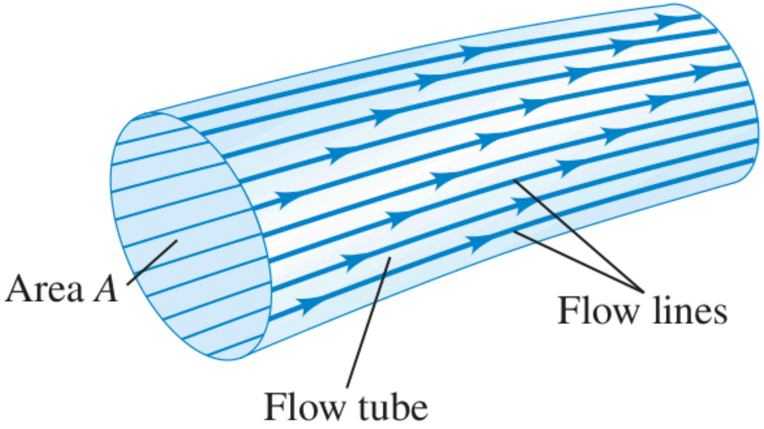


Moving fluids

- Liquids are approximately incompressible in most situations 💦
- Gases may also be treated as incompressible if the pressure differences from one region to another are not too great 🌬️
- Viscosity is how thick or sticky a fluid is, and how much it resists flowing. It's an internal friction that makes it harder for the different parts of fluid to slide past each other (shear stress)

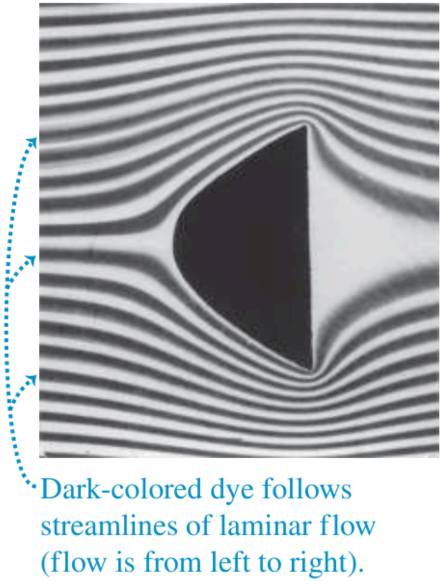
Fluid flow

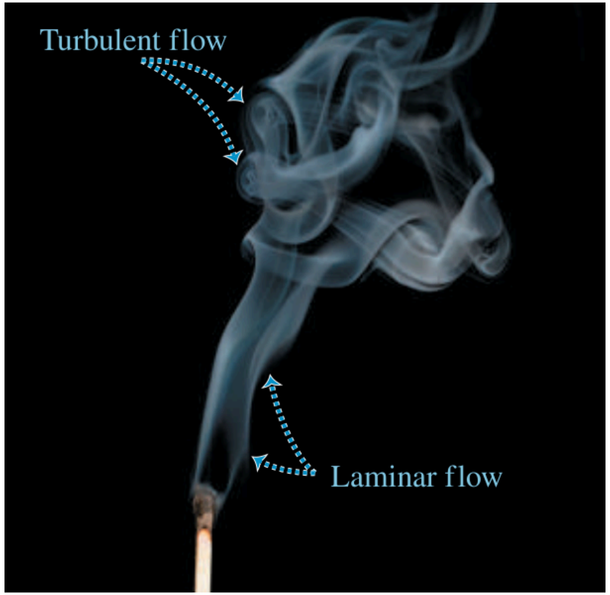
- The path of an individual particle in a moving fluid is called a **flow line**
- A **streamline** is a curve tangent to the fluid velocity vector at that point
- A **flow tube** is a tube bounded at its sides by flow lines



- In **steady flow**, the overall flow pattern does not change with time, so every element passing thru follows the same flow line
  - Here, fluid cannot cross the walls of a flow tube

- If the adjacent layers of fluid slide smoothly past each other and the flow is steady, we have a **laminar flow**
  - eg. Photo shows fluid flow around an obstacle, made by injecting dye into water





- At sufficiently high flow rates or when boundary surfaces cause abrupt changes in velocity, flow can be irregular and chaotic. This is called **turbulent flow**

The continuity equation

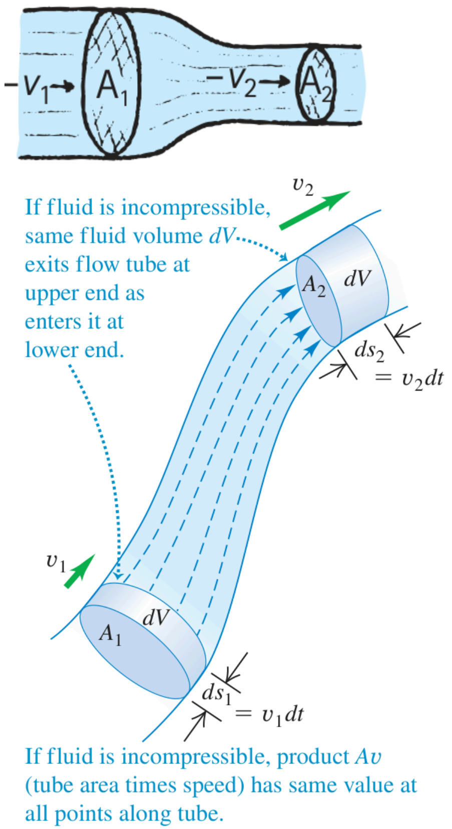
- The mass of a moving fluid doesn't change as it flows. This conservation of mass is described by the **continuity equation**

$$A_1 v_1 = A_2 v_2$$

which relates the flow speeds v_1 and v_2 for two cross sections A_1 and A_2 in a flow tube

- For fluids that are *not* incompressible,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$



- The product Av is equal to the rate at which a volume of fluid crosses a section of the tube. This is the **volume flow rate**, sometimes called **flux** Q:

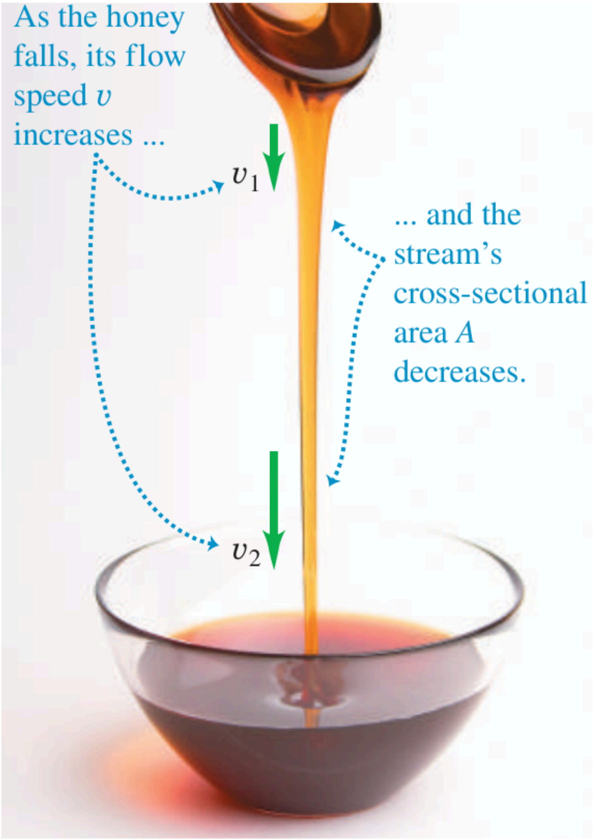
$$Q = \frac{\Delta V}{\Delta t} = Av$$

which means that the continuity equation is also $Q_1 = Q_2$

- Its SI unit is m^3/s. But other units like L/min for blood flow are commonly used
    - Btw, a liter (L) is 1/1000 of a cubic meter (10^{-3} m^3) or 1000 cubic centimeters (10^3 cm^3)

Example. The continuity equation helps explain the shape of a stream of honey poured from a spoon

- It shows that the volume flow rate has the same value at all points along any flow tube
- And so, when the cross section of a flow tube decreases, the speed increases, and vice versa



The volume flow rate $dV/dt = Av$ remains constant.

Example. How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

- Flux equation gives us $\Delta V = Q \times \Delta t$. And so

$$\Delta V = \frac{5\ \mathrm{L}}{1\ \mathrm{min}} \times \left[75\ \mathrm{yr} \times \frac{365\ \mathrm{dy}}{1\ \mathrm{yr}} \times \frac{24\ \mathrm{hr}}{1\ \mathrm{dy}} \times \frac{60\ \mathrm{min}}{1\ \mathrm{hr}}\right] \times \frac{10^{-3}\ \mathrm{m}^3}{1\ \mathrm{L}}$$

$$= 2 \times 10^5\ \mathrm{m}^3$$

- This amount is about 200,000 tons of blood. For comparison, this is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap swimmning pool 🏊

Example. A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

- For (a), we find $Q = Av$ and note that the cross-sectional area is $A = \pi r^2$, giving us

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2} = \frac{(0.5 \text{ L/s}) \times (10^{-3} \text{ m}^3/\text{L})}{\pi (9 \times 10^{-3} \text{ m})^2} = 1.96 \text{ m/s}$$

- For (b), we could repeat the calculation, but we'll use the continuity equation instead

$$A_1 v_1 = A_2 v_2$$

which by isolating v_2 gives

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_1^2}{r_2^2} v_1 = \frac{(0.9\ \text{cm})^2}{(0.25\ \text{cm})^2} \times 1.96\ \text{m/s} = 25.5\ \text{m/s}$$

- A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube 🪴💦

Bernoulli's equation

Bernoulli's principle

- From the garden hose 🪴💦 example above, we observe that the narrower the flow tube, the faster the flow speed
  - This is a consequence of **principle of continuity**, as in water being a continuous flow, hence obeying continuity equation
- For flow to be continuous in a confined region, it speeds up when moving from a wider region to a narrower one

- This is more generally known as **Bernoulli's principle**, which can be stated as where the speed of a fluid increases, the internal pressure in the fluid decreases
  - And vice versa: where speed decreases, pressure increases. This applies when friction, turbulence, and changes in height don't affect pressure; and for fluids in *steady flow*

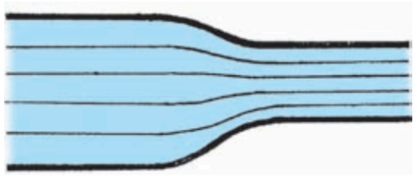


FIGURE 14.17
Water speeds up when it flows into the narrower pipe. The close-together streamlines indicate increased speed and decreased internal pressure.

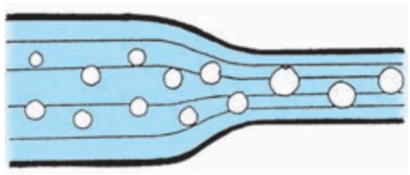
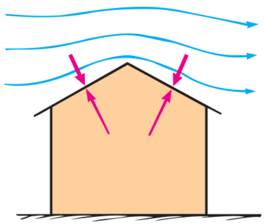
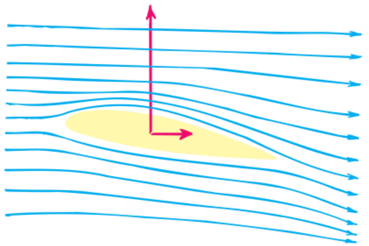
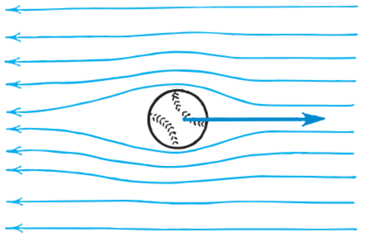
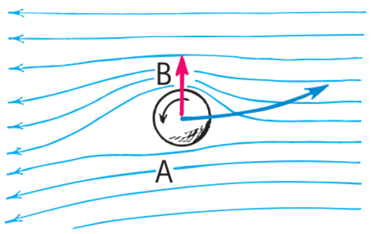


FIGURE 14.18
Internal pressure is higher in slower-moving water in the wide part of the pipe, as evidenced by the more-squeezed air bubbles. The bubbles are bigger in the narrow part because the internal pressure there is lower.



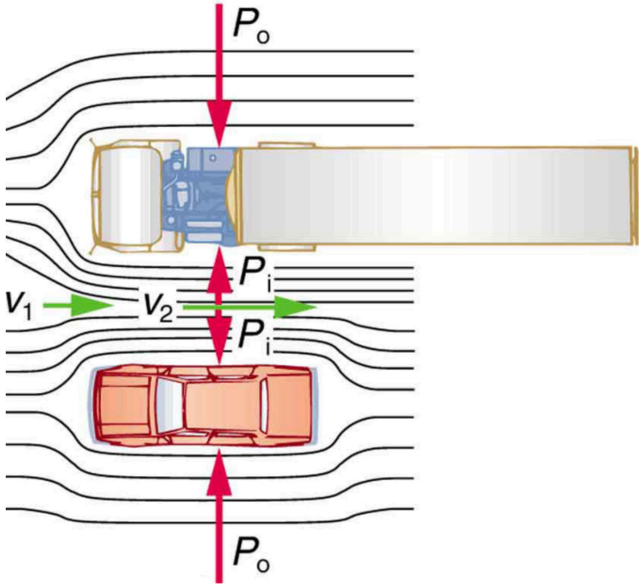






Example.

- The air pressure above the roof is lower than the air pressure beneath the roof 🏠
- The vertical vector represents the net upward force called *lift* that results from higher air pressure below the wing than above the wing. The horizontal vector represents *air drag*
- A non-spinning vs. spinning baseball. The latter produces a crowding of streamlines. The resulting lift (red arrow) causes the ball to curve (blue) ⚾

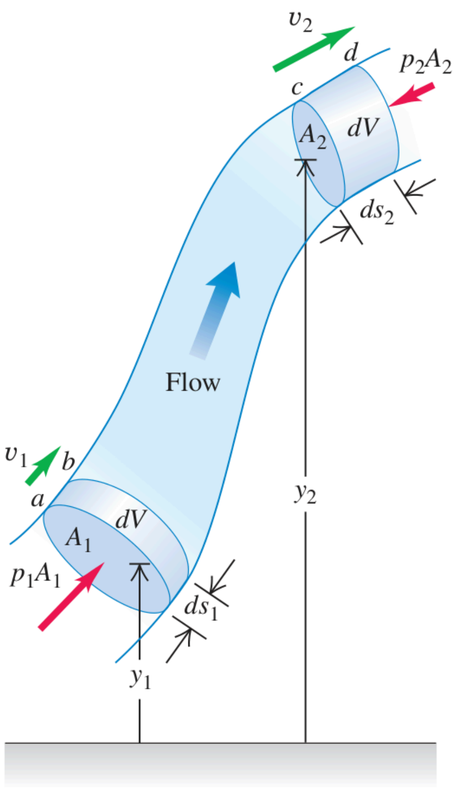


Example. An overhead view of a car passing a truck on a highway. The air passing between the vehicles flows in a narrower channel and must increase its speed ($v_2 > v_1$), causing the pressure between them to drop ($p_i < p_o$)

- Greater pressure on the outside pushes car and truck together
- There's low pressure area (LPA) between the car and truck 🌀
- We can see that Bernoulli's can be mathematically formalized...

Deriving Bernoulli's equation

- Following from the continuity equation, when an incompressible fluid flows along a flow tube with varying cross section, its speed *must* change, and so an element of fluid must have acceleration
  - If the tube is horizontal, the force that causes this acceleration has to be applied by the surrounding fluid. This means that pressure must be different in regions of different cross section
  - If the elevation changes, this also causes a pressure difference



- Using the concepts of work W, kinetic energy K, and potential energy U, we relate pressure, flow speed, and height for flow of a fluid
- The net work done on a fluid element by the pressure of the surrounding fluid equals the change in kinetic energy plus the change in gravitational potential energy:

$$\Delta W = \Delta K + \Delta U$$

$$(p_1 - p_2)\Delta V = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2) + \rho \Delta V g(y_2 - y_1)$$

Bernoulli's equation

- **Bernoulli's equation** states that a quantity involving the pressure p, flow speed v, and elevation y has the same value anywhere in a flow tube, assuming steady flow in an ideal fluid
  - This equation can be used to relate the properties of the flow at any two points

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

$$\text{or} \qquad p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

- Note that when the fluid is *not* moving, such that $v_1 = v_2 = 0$, this reduces to to the pressure-depth relationship that we discussed in fluid statics lecture:

$$p_1 = p_2 + \rho g(y_2 - y_1)$$

- Note again that Bernoulli's equation is valid for only incompressible, steady flow of a fluid with no viscosity

Example. In the previous garden hose 🪴💦 example, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that absolute pressure in nozzle is (atmospheric, as it must be) and assuming level, frictionless flow.

- Isolating pressure p_1 in the hose gives us

$$p_1 = p_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$
\begin{aligned}
p_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\
&\qquad + \frac{1}{2}(10^3 \text{ kg/m}^3)\left[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2\right] \\
&= 4.24 \times 10^5 \text{ N/m}^2
\end{aligned}
$$

- This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure p_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions

Example. Healthy giraffes have high blood pressure

- Bernoulli's equation suggests that as blood flows upward at roughly constant speed v from the heart to the brain, the pressure p will drop as the blood's height y increases

- For blood to reach the brain with required minimal pressure, the human heart provides a maximum (systolic) gauge pressure of about 120 mm Hg. The vertical distance from heart to brain is much larger for a giraffe, so its heart must produce a much greater maximum gauge pressure (about 280 mm Hg)

Viscosity and turbulence

Viscosity

- **Viscosity** is internal friction in a fluid
- Viscous forces oppose motion of one portion of a fluid relative to another
- It is the reason it takes effort to paddle a canoe through calm water, but it is also the reason the paddle works
- Viscous effects are important in the flow of fluids in pipes, flow of blood, lubrication of engine parts, and many other situations

- Fluids that flow readily, such as water or gasoline, have smaller viscosities than do "thick" liquids such as honey or motor oil
- Viscosity is strongly *temperature dependent*, increasing for gases and decreasing for liquids as temperature increases
  - eg. lava 🌋 is viscous – its viscosity decreases with increasing temperature. The hotter it is, the more easily it can flow

Turbulence

- When the speed of a flowing fluid exceeds a certain critical value, the flow is no longer *laminar*
- Instead, the flow pattern becomes extremely irregular and complex, and it changes continuously with time. There is no steady-state pattern
- This irregular, chaotic flow is called **turbulence**

- Bernoulli's equation is not applicable to regions where turbulence occurs because the flow is not steady

- Whether a flow is laminar or turbulent depends in part on viscosity. The greater the viscosity, the greater the tendency for the fluid to flow in sheets (laminae) and the more likely the flow is to be laminar. In fact, a *little* viscosity is needed to ensure that the flow is laminar

    - eg. the ketchup for your fries 🍟 is viscous and thus more likely to have a laminar flow

Example. Listening for turbulent flow

- Normal blood flow in the human aorta is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent
- Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique

Quiz time 🕐

Blood flow 🩸

Blood flows through a coronary artery that is partially blocked by deposits along the artery wall. Through which part of the artery is the flow speed largest?

- narrow part
- wide part
- flow speed is the same in both parts