Fluid statics

R. Torres

2025 W47[1]

[1]Phys 20.01 Mod 5. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Fluids and density

Pressure in a fluid

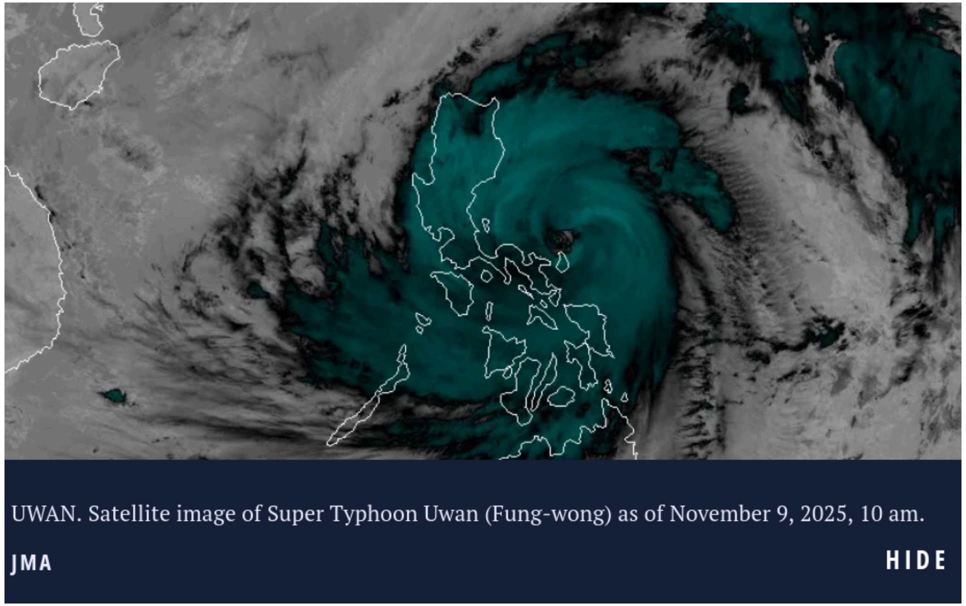
Buoyancy

Quiz time 🕐

Fluids and density

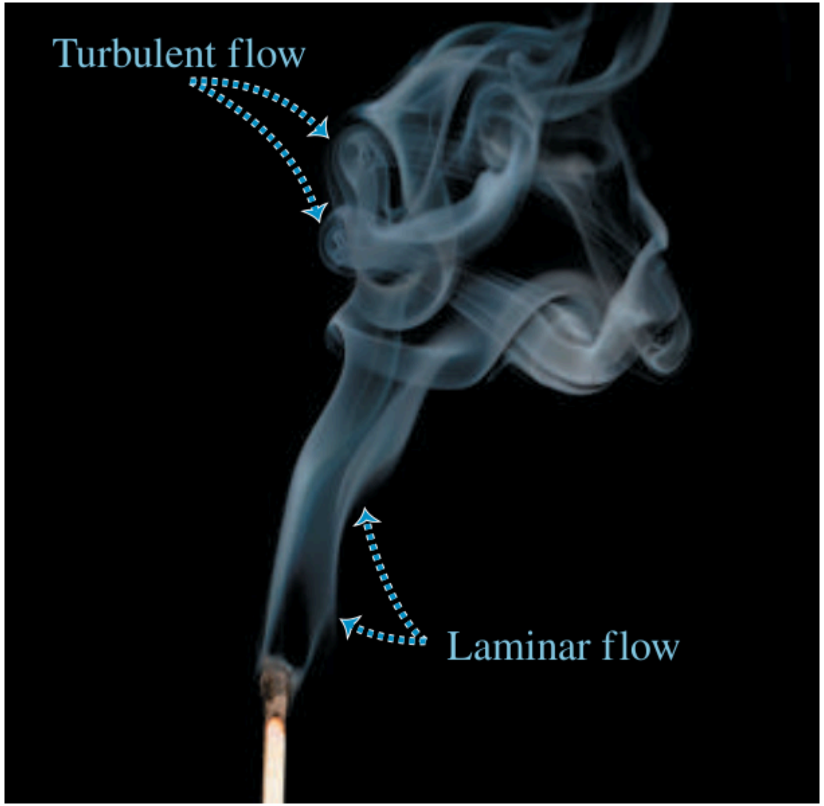
Fluids on the daily

- Fluids play a vital role in many aspects of everyday life
- We drink, breathe, swim in them. They circulate our bodies, control weather
- **Fluid statics**, the study of fluids at rest in equilibrium, is based on Newton's first and third laws



Fluids on the daily

- **Fluid dynamics**, the study of fluids in motion, is much more complex – one of the most complex in physics
- Fortunately, we can analyze many situations by using simple models and familiar principles like Newton's laws and conservation of energy



Fluids

- A **fluid** is any substance that can flow and change the shape of the volume that it occupies
  - By contrast, a solid tends to maintain its shape
- We use the term “fluid” for both gases and liquids
  - Key difference is that liquid has cohesion, while gas does not
  - Molecules in a liquid are close to one another, so they attract each other and tend to stay together ie. to cohere (cf. gas)
  - That’s why a quantity of liquid maintains the same volume as it flows, while gases change volume quickly by expanding

Density

- An important property of any material, fluid or solid, is its **density** ρ. Think of it as a measure of compactness
- For a homogeneous material of mass m occupying volume V:

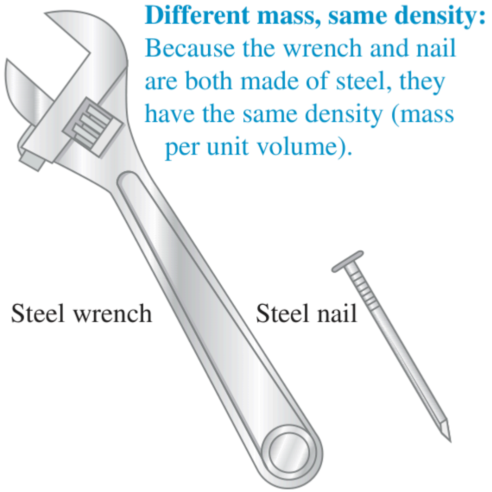
$$\rho = \frac{m}{V}$$

  - Homogeneous means it has same density throughout, eg. ice
- For non-homogeneous ones, ρ above describes **average density**
  - eg. human body with its low-density fats, high-density bones

- Its SI unit is kilogram per cubic meter ($1\ kg/m^3$). The cgs unit, the gram per cubic centimeter ($1\ g/cm^3$) is also widely used

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerin	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.



- The **specific gravity** of a material is the ratio of its density to density of water (at $4°C, 1000\ kg/m^3$), eg. aluminum has 2.7

Example.

a. (Easy) When water freezes, it expands. What does this say about the density of ice compared with the density of water?

b. (Tricky) Which weighs more: a liter of ice or a liter of water?

c. Which has greater density: 100 kg lead or 1000 kg aluminum?

d. What is the density of 1000 kg of water?

e. What is the volume of 1000 kg of water?

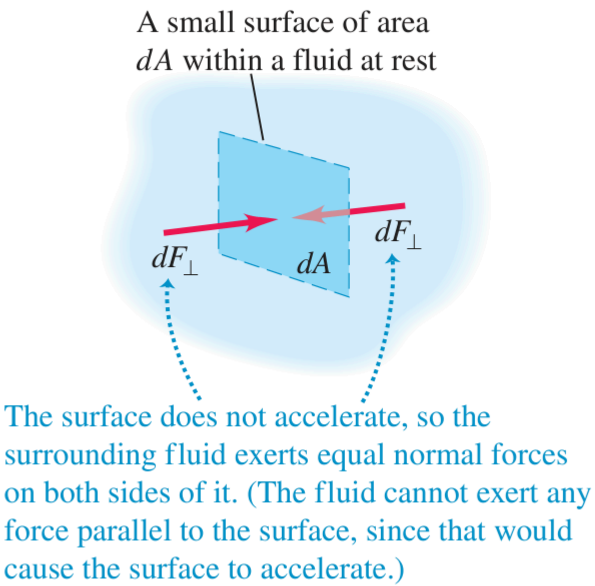
- Ice is less dense than water (because it has more volume for the same mass), which is why ice floats on water
- A liter of water weighs more. Ice is less dense than water as water expands when it freezes. Therefore, a liter of ice will have less mass packed into that volume compared to a liter of water
- Density is a ratio of mass to volume so it is the same for any mass. Thus and as per table, lead is more dense ($\rho_{\mathrm{Pb}} > \rho_{\mathrm{Al}}$)
- The density of any amount of water is 1000 $\mathrm{kg/m^3}$ (or 1 $\mathrm{g/cm^3}$)
- The volume of 1000 kg of water is 1 $\mathrm{m^3}$

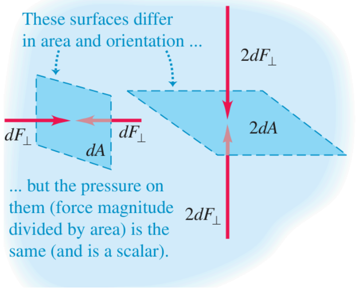
Pressure in a fluid

Pressure

- A fluid exerts force perpendicular to any surface in contact with it
  - eg. force you feel pressing on your legs when in pool
- **Pressure** is the normal force exerted by fluid on an area:

$$p = \frac{F_\perp}{A}$$

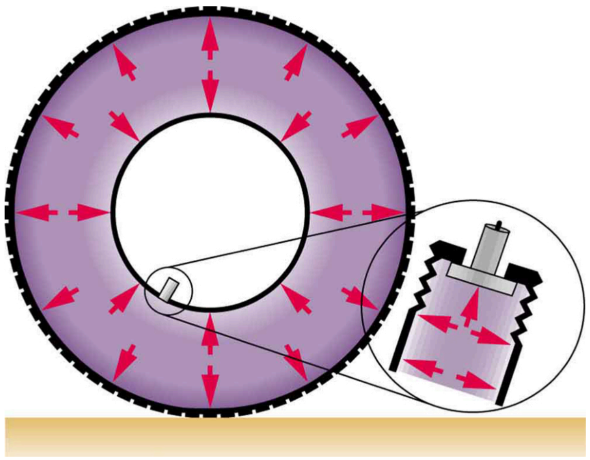


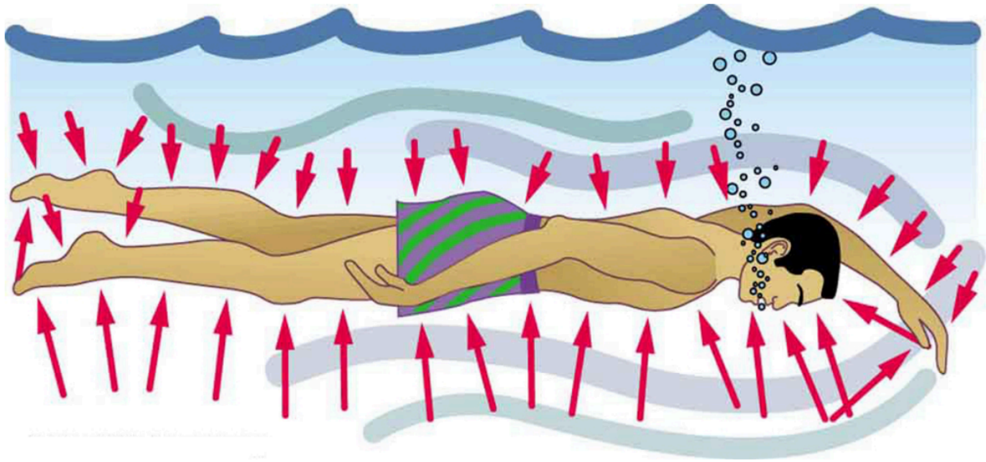


Pressure

- Its SI unit is pascal (Pa): 1 Pa = 1 $\mathrm{N/m^2}$
- **Atmospheric pressure** p_a is the pressure of earth's atmosphere, the pressure at the bottom of this sea of air in which we live. It varies with weather changes and with elevation
  - Normal p_a at sea level is 1 atmosphere (atm): 1 atm $= 101,325$ Pa $= 1.013 \times 10^5$ Pa
- Btw, don't confuse pressure (a scalar) and force (a vector)
  - Pressure acts perpendicular to any surface, no matter how that surface is oriented, & has no direction of its own (scalar)

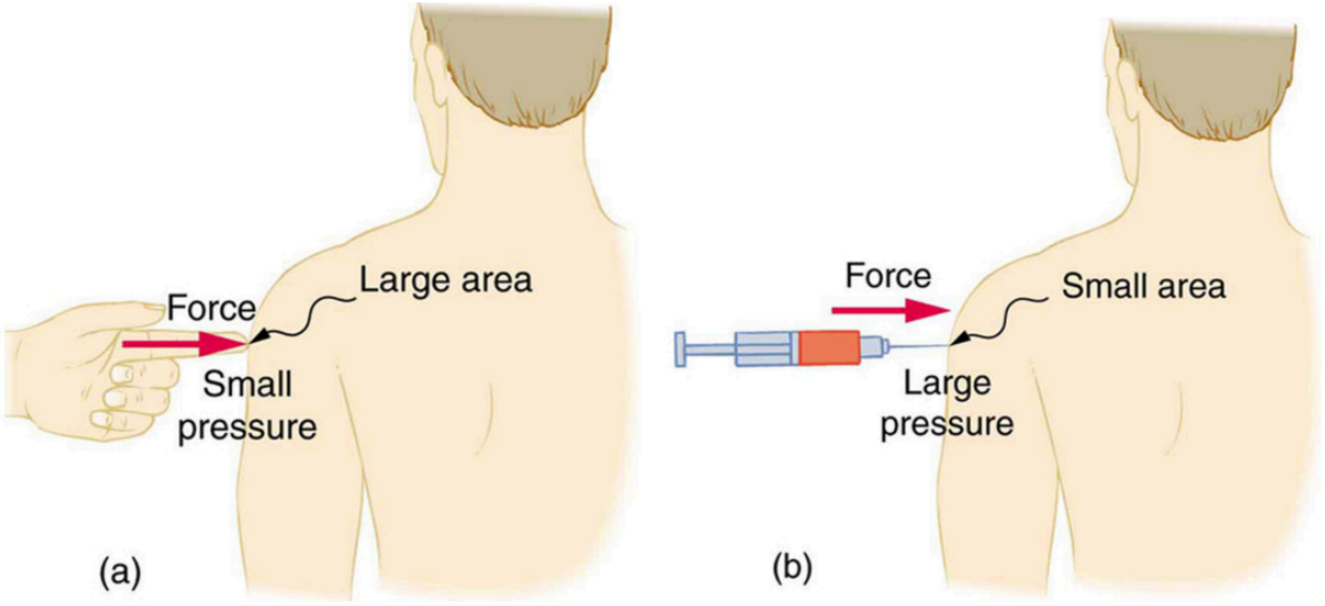
Example. Pressure inside this tire exerts forces perpendicular to all surfaces it contacts





Example. Pressure is exerted on all sides of this swimmer, since water would flow into the space he occupies if he were not there. Note the larger (buoyant) forces underneath due to greater depth

Example. While the person being poked with the finger might be irritated, the force has little lasting effect. In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.



Example. Given a living room with a 4 m $\times$ 5 m floor and a ceiling 3.0 m high, what is the total downward force on the floor due to the air pressure of 1.00 atm?

- The floor is horizontal, so $F_\perp$ is vertical (downward)
- We have $A = (4 \text{ m})(5 \text{ m}) = 20 \text{ m}^2$. So, the downward force is

$$F_\perp = pA = \left(1.013 \times 10^5 \text{ N/m}^2\right)\left(20 \text{ m}^2\right) = 2.0 \times 10^6 \text{ N}$$

Pressure and depth

- In fluids, pressure variations are important
  - eg. Atmospheric pressure is less at high altitude than at sea level, which is why airliner cabins have to be pressurized ✈️
  - eg. When you dive into deep water, you can feel the increased pressure on your ears 🏊
- As we move upward in the fluid, pressure decreases

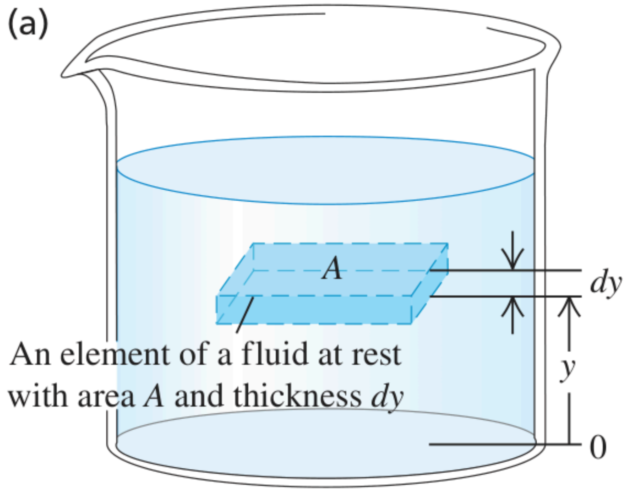
- If p_1 and p_2 are the pressures at elevations y_1 and y_2, then the pressure difference between points 1 and 2 in a static fluid of uniform density ρ (an incompressible fluid) is proportional to the difference between the elevations y_1 and y_2:

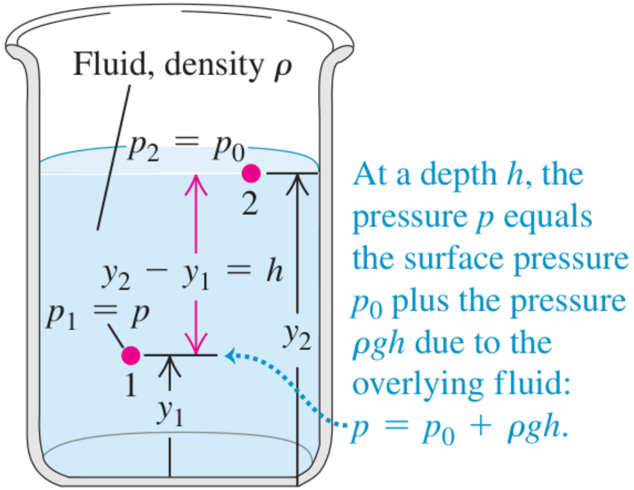
$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

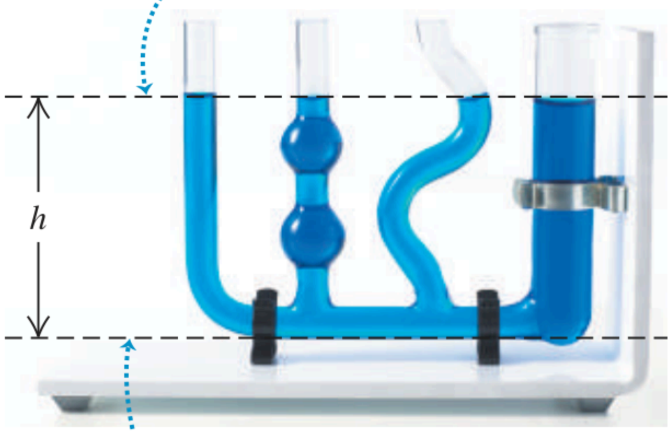
- It's often convenient to express this in terms of depth h below the surface. Given p_0 is pressure at surface, pressure below is

$$p = p_0 + \rho g h$$

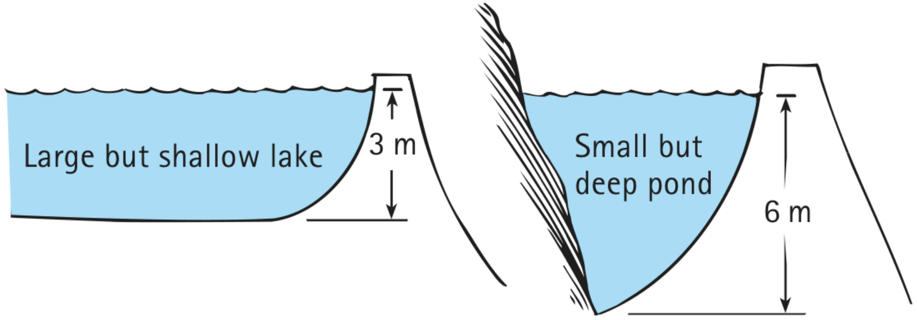
- Observe: similar to gravitational potential energy $U_g = mgh$

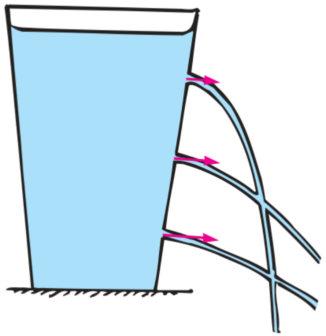

(a)
A
dy
y
0
An element of a fluid at rest with area A and thickness dy


Fluid, density ρ
$p_2 = p_0$
2
$y_2 - y_1 = h$
$p_1 = p$
1
y_1
y_2
At a depth h, the pressure p equals the surface pressure p_0 plus the pressure $\rho g h$ due to the overlying fluid: $p = p_0 + \rho g h$.
Pressure difference between levels 1 and 2: $p_2 - p_1 = -\rho g(y_2 - y_1)$
The pressure is greater at the lower level.


The pressure at the top of each liquid column is atmospheric pressure, p_0.
h
The pressure at the bottom of each liquid column has the same value p.
The difference between p and p_0 is $\rho g h$, where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

Example. Pressure does not depend on the amount of liquid present. Volume is not the key, depth is. Here, the large, shallow lake exerts only 1/2 the pressure that the small, deep pond exerts

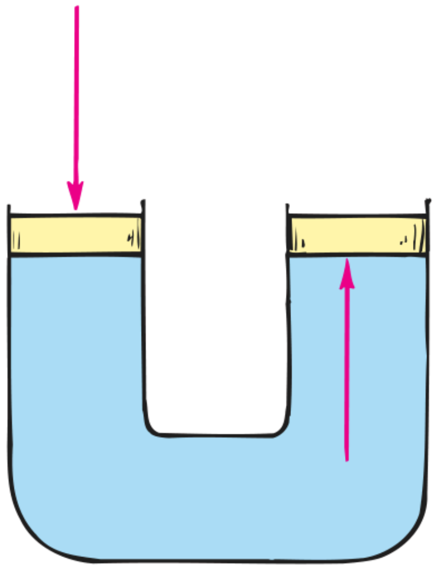


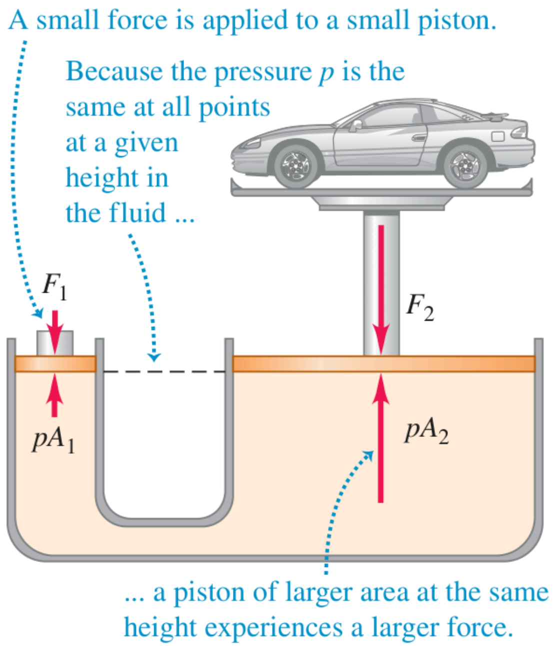


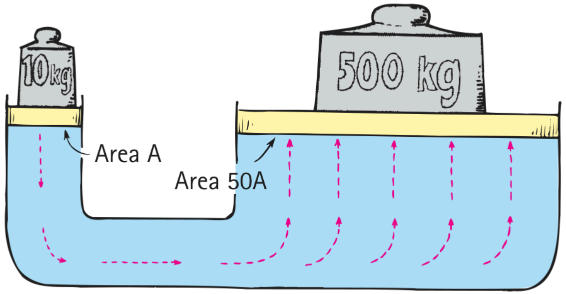
Example. The pressure (and consequently, the force acting perpendicularly to the walls of the cup's inner surface) increases with increasing depth

Pascal's law

- The above equation (obtained via pressure and depth) shows that a change in pressure at one part of the fluid is transmitted undiminished to all other parts of the fluid. This observation is called Pascal's law
  - **Pascal's law** states that pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid





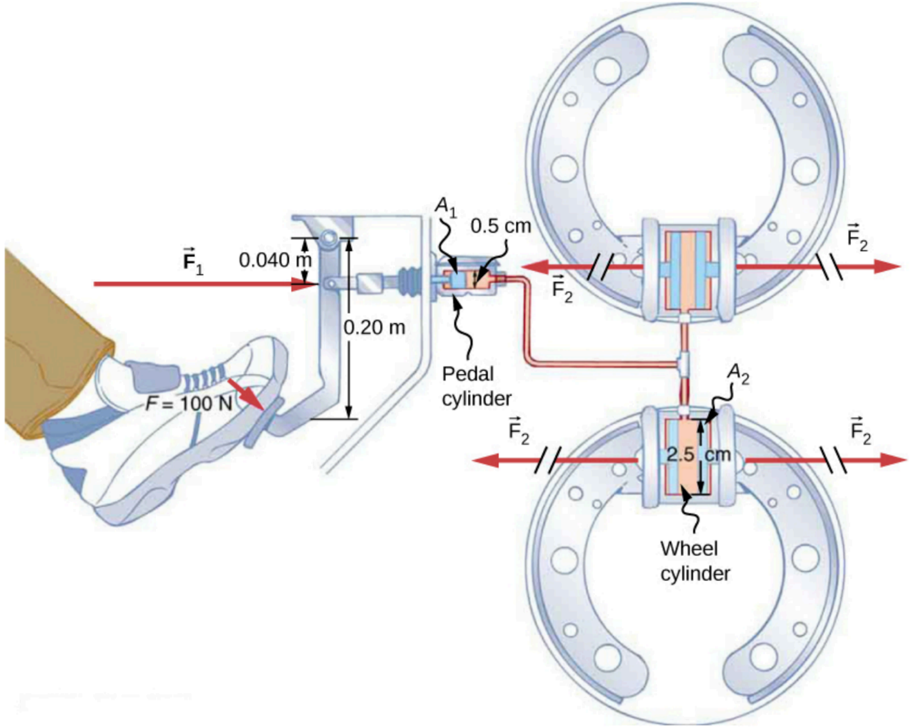


- The hydraulic lift illustrates this law. The applied pressure is same in both pistons, so

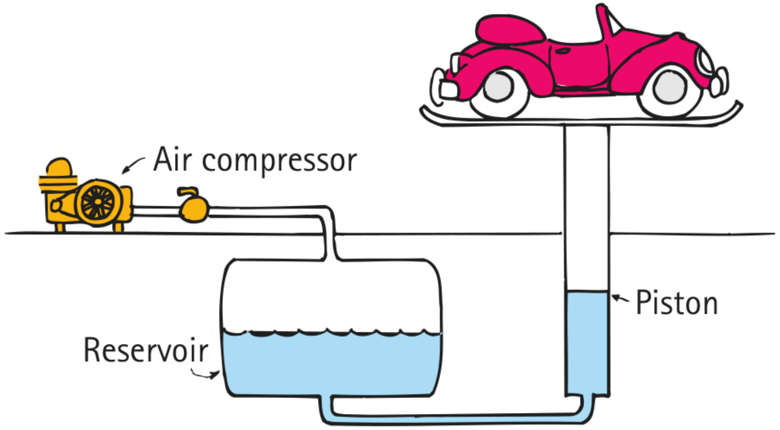
$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad \text{and} \qquad F_2 = \frac{A_2}{A_1} F_1$$

- This lift is a force-multiplying device. Dentist's chairs, car lifts and jacks, many elevators, and hydraulic brakes all use Pascal's law

Example. Hydraulic brakes. The driver exerts a force of 100 N on the brake pedal. This force is increased by simple lever and again by hydraulic system. Each of the identical wheel cylinders receives the same pressure and therefore creates the same force output F_2



Example. As the car is being lifted, how does the oil level change in the reservoir compare with the distance the car moves?



- The car moves up a greater distance than the oil level drops, since the area of the piston is smaller than the surface area of the oil in the reservoir

Absolute pressure and gauge pressure

- **Absolute pressure** is the total pressure in a fluid. **Gauge pressure** is the difference between absolute pressure and atmospheric pressure
- eg. If the pressure inside a car tire is equal to atmospheric pressure, then the tire is flat
  - The pressure inside has to be greater to support the car
  - Key quantity is difference between inside & outside pressures
  - Here, excess pressure above atmospheric is the gauge pressure

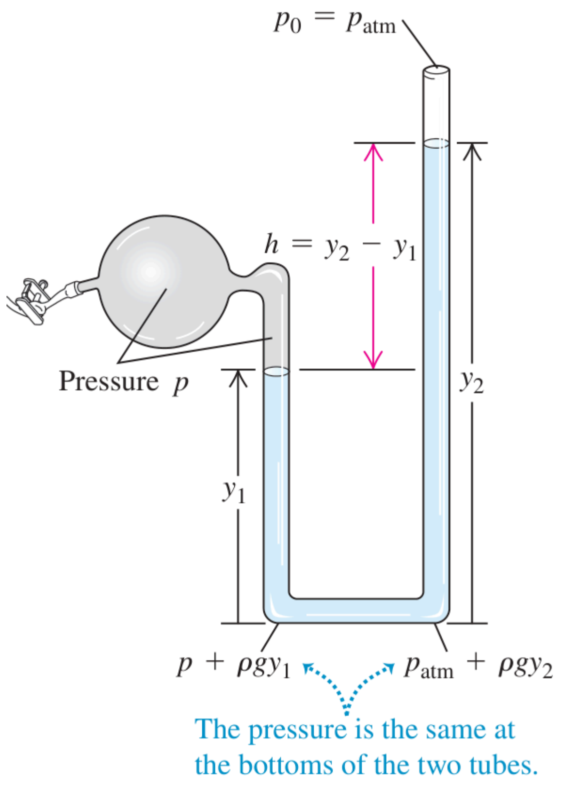
Example. When we say that the pressure in a car tire is 32 pounds (actually 32 lb/in^2), we mean that it is greater than atmospheric pressure (1 atm) by this amount. The total (absolute) pressure in the tire is then 47 lb/in^2 because 1 atm $= 14.7\ \text{lb/in}^2$ and

$$\begin{aligned} \text{absolute pressure} &= \text{gauge pressure} + \text{atmospheric pressure} \\ &= (32 + 14.7)\ \text{lb/in}^2 \approx 47\ \text{lb/in}^2 \end{aligned}$$

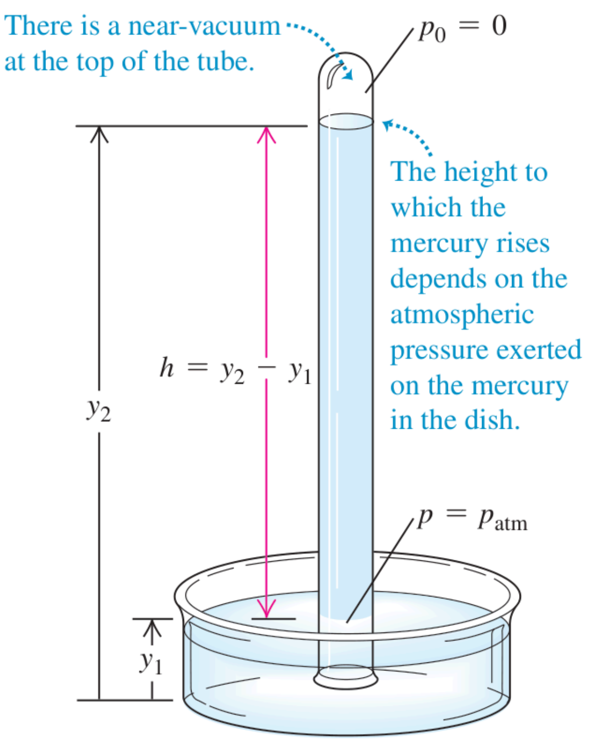
- Btw, engineers use the abbreviations psig and psia for "pounds per square inch gauge" and "pounds per square inch absolute"

Pressure gauges

- Here, gauge means measure
- Simplest pressure gauge is the open-tube **manometer**
  - U-shaped tube contains liquid, often Hg or H2O
  - Left tube is connected to the container where the pressure is measured, while right is open to the atmosphere



- Another common pressure gauge is the **mercury barometer**
  - Long tube, closed at one end, filled with Hg, then inverted in a dish of Hg
  - Space above Hg column contains only Hg vapor. Its pressure is negligibly small, so pressure at top of column is practically zero



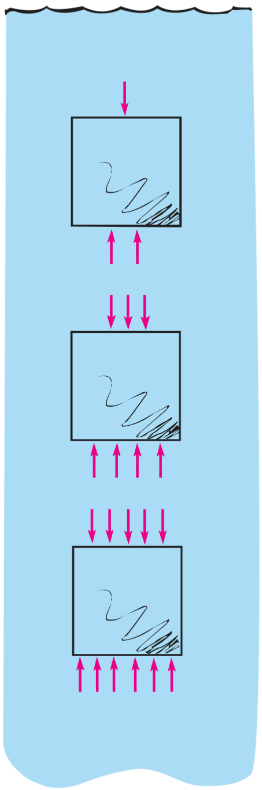
Example. Gauge pressure of blood

- Blood-pressure readings, such as 130/80, give the maximum and minimum gauge pressures in the arteries, measured in mm Hg or torr
- Blood pressure varies with vertical position within the body. The standard reference point is upper arm, level with the heart
- Btw, the device (sphygmo*manometer*) used to measure blood pressure is a pressure gauge of the first type (*manometer*)

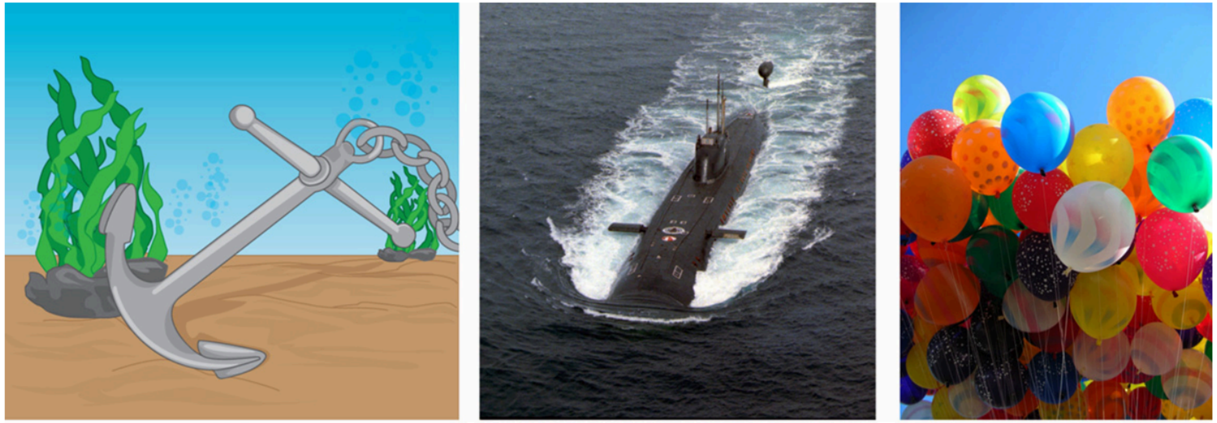
Buoyancy

What's buoyant?

- Lifting something heavy underwater is relatively easy compared to lifting the same thing above surface. 🤔
- How? When the heavy thing is submerged, the water exerts an upward force on it that is exactly opposite to the direction of gravity's pull (weight)
- This upward force is **buoyant force**, a result of pressure increasing with depth



Example. There is **buoyancy** when buoyant forces are present. (a) Even objects that sink, like an anchor, are partly supported by water when submerged ⚓. (b) Submarines have adjustable density via ballast tanks so they may float/sink as desired 🚢. (c) Helium-filled balloons tug upward, showing air's buoyant effect 🎈

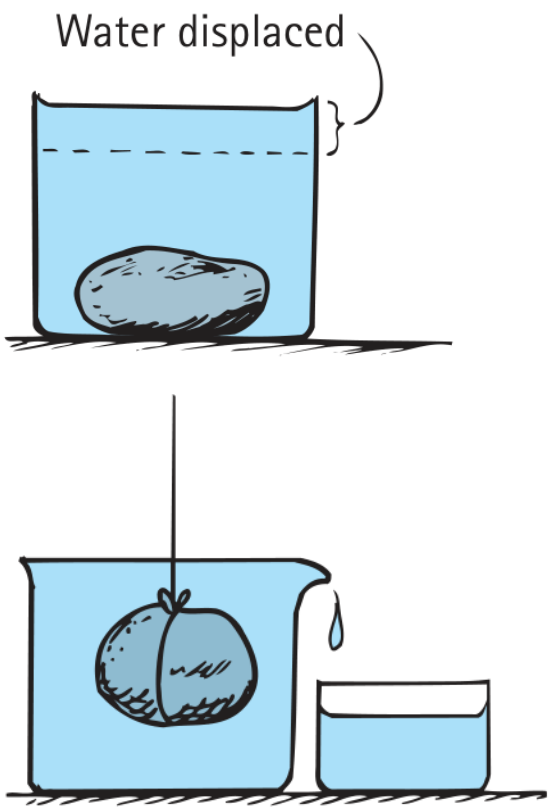


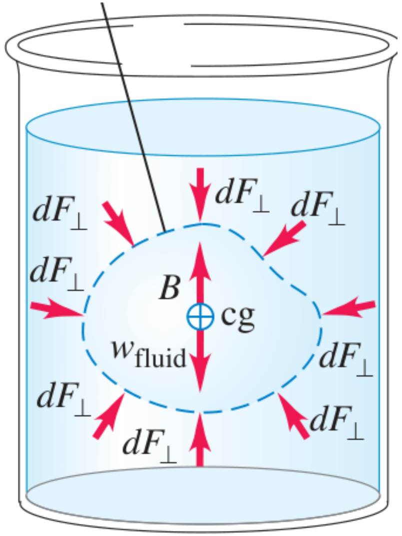
(a) (b) (c)

Archimedes's principle

- Buoyancy is described by **Archimedes's principle**, which states that when an object is immersed in a fluid, the fluid exerts an upward buoyant force B on object equal to the weight w_{fluid} of fluid that the object displaces

$$\vec{B} = -\vec{w}_{\text{fluid}} \quad \text{or} \quad B = w_{\text{fluid}}$$





Buoyant force depends on fluid density

- Buoyant force is proportional to the density of the *fluid* in which the object is immersed, *not* the density of the object
    - eg. If a wooden block and an iron block have the same volume and both are submerged in water, both experience the same buoyant force
    - The wooden block rises and the iron block sinks because this buoyant force is greater than the weight of the wooden block but less than the weight of the iron block

Example.

a. True or false? Archimedes's principle tells us that any object that displaces 10 N of liquid will be buoyed up with 10 N.

b. A 1-L container filled with lead has a mass of 11.3 kg and is submerged in water. What is the buoyant force acting on it?

c. As a boulder thrown into a deep lake sinks deeper and deeper into water, does the buoyant force on it increase or decrease?

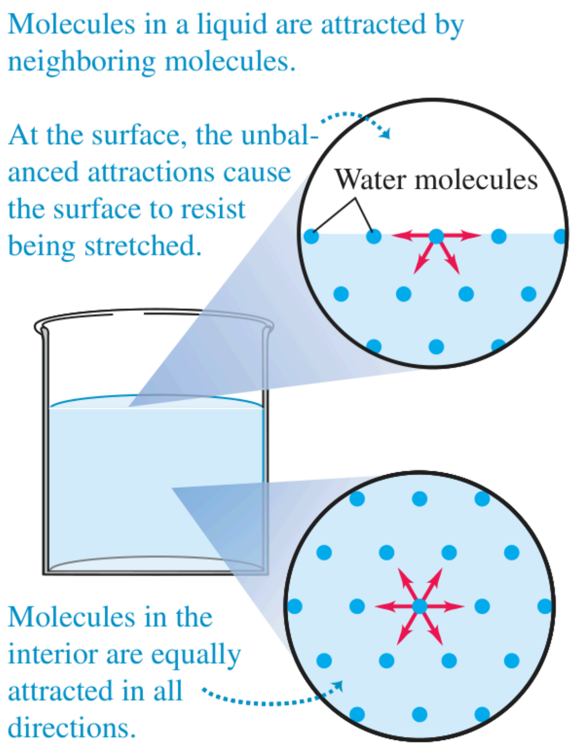
- True. It's only the weight of displaced liquid that counts ⚖️💧
- The buoyant force equals weight of the liter of water displaced – not the weight of lead. 1 L of water has a mass of 1 kg and weighs ~10 N, so the buoyant force on it is 10 N
- The buoyant force remains unchanged as the boulder sinks because the boulder displaces the same volume and the same weight of water at any depth 🪨

What makes an object sink or float?

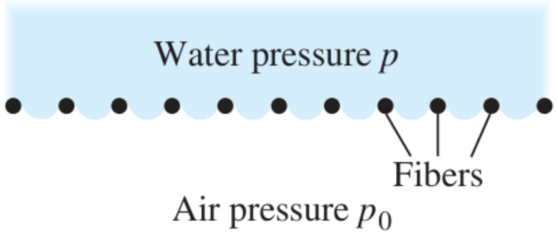
- An object more dense than fluid in which it is immersed sinks
- An object less dense than fluid in which it is immersed floats
- An object that has a density equal to the density of the fluid in which it is immersed neither sinks nor floats
- **But**, a paperclip can rest atop a water surface even though its density is several times that of water 📎. This is an example of surface tension

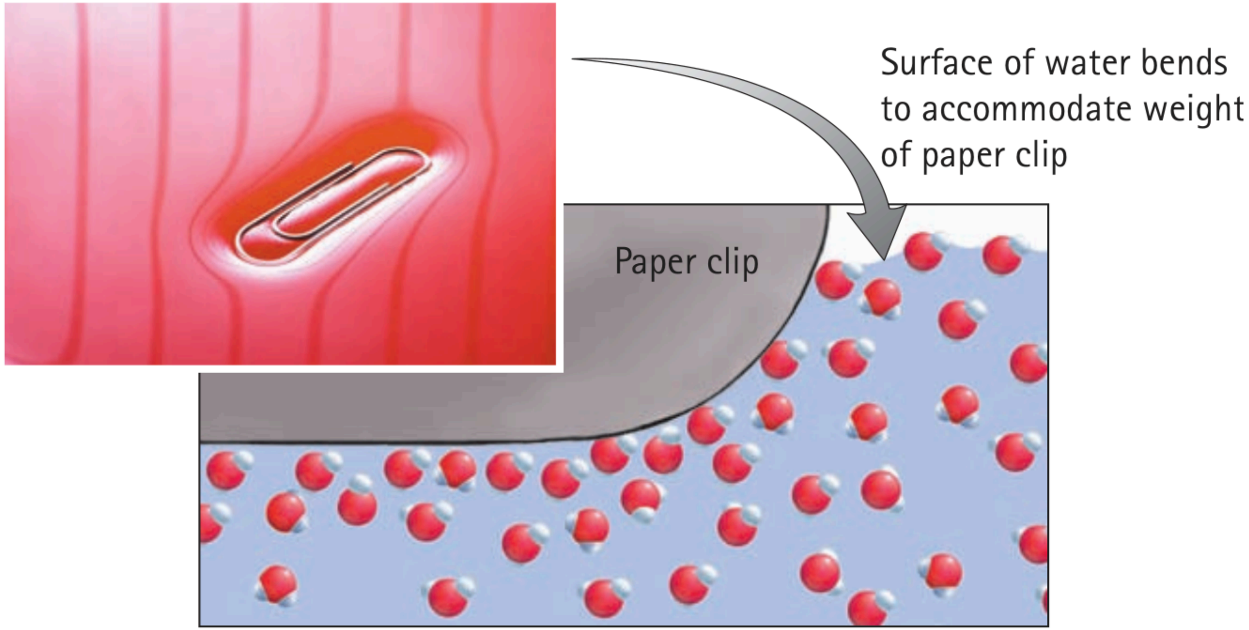
Surface tension

- The surface of the liquid behaves like a membrane under tension
- **Surface tension** arises because the molecules of the liquid exert attractive forces on each other
  - Liquid tends to minimize its surface area, just as stretched membrane does



- Surface tension makes it difficult to force water through small crevices. The required water pressure p can be reduced by using hot, soapy water, which has less surface tension





Quiz time 🕐

Blood pressure and intravenous infusions 💉

Intravenous infusions are usually made with the help of gravity. Assuming that density of the fluid being administered is 1.00 g/mL, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if blood pressure in the vein is 18 mm Hg above atmospheric pressure?

Conversion to N/m^2 (Pa)	Conversion from atm
1.0 atm = 1.013×10^5 N/m^2	1.0 atm = 1.013×10^5 N/m^2
1.0 $dyne/cm^2$ = 0.10 N/m^2	1.0 atm = 1.013×10^6 $dyne/cm^2$
1.0 kg/cm^2 = 9.8×10^4 N/m^2	1.0 atm = 1.013 kg/cm^2
1.0 $lb/in.^2$ = 6.90×10^3 N/m^2	1.0 atm = 14.7 $lb/in.^2$
1.0 mm Hg = 133 N/m^2	1.0 atm = 760 mm Hg
1.0 cm Hg = 1.33×10^3 N/m^2	1.0 atm = 76.0 cm Hg
1.0 cm water = 98.1 N/m^2	1.0 atm = 1.03×10^3 cm water
1.0 bar = 1.000×10^5 N/m^2	1.0 atm = 1.013 bar
1.0 millibar = 1.000×10^2 N/m^2	1.0 atm = 1013 millibar