Forces and laws of motion: specific applications

R. Torres

2025 W38[1]

[1]Phys 20.01 Mod 2. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously ⏪

Friction forces 🐌

Elasticity 🪱

Gravitation 🪐

Quiz time 🕐

Previously ⏪

Applications of forces and laws of motion

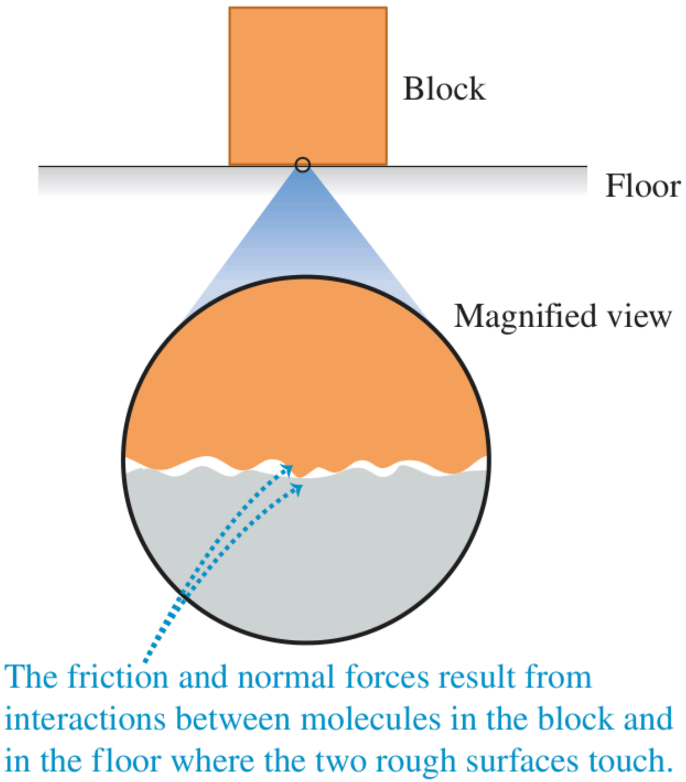
Friction forces 🐌

Friction

- Friction is important in many aspects of everyday life
  - eg. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car 🚗
- Air drag, the friction force exerted by the air on an object moving through it, decreases automotive fuel economy but makes parachutes work 🪂

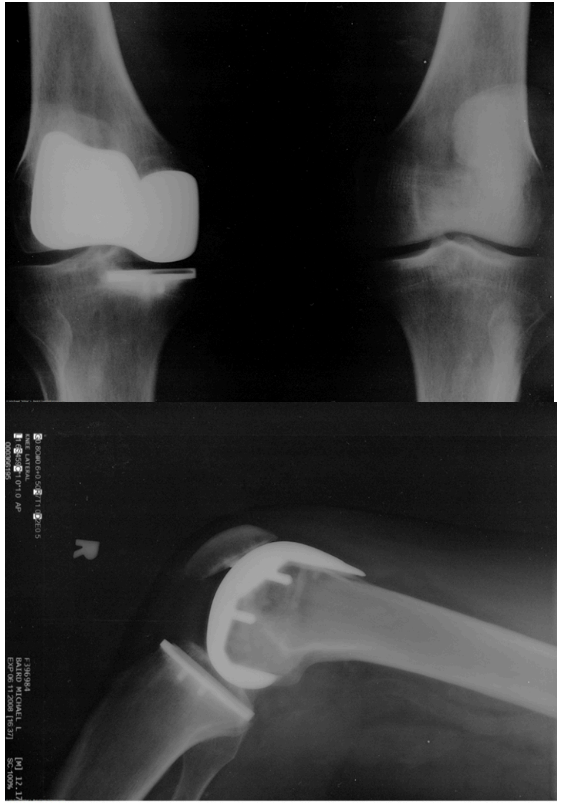
Friction

- On a microscopic level, friction and normal forces result from intermolecular forces (electrical in nature) between two rough surfaces at points where they come into contact



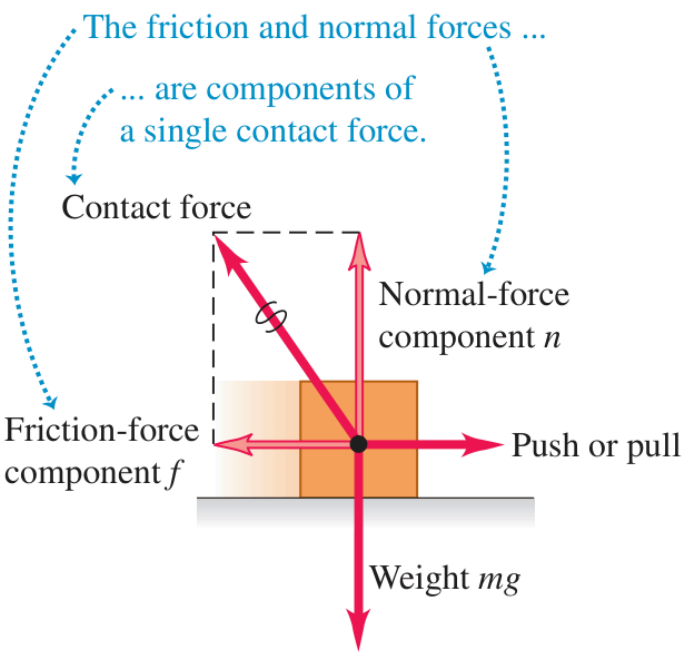
Friction in body

- The joints produce synovial fluid that reduces friction and wear between ends of two bones
- A damaged or arthritic joint can be replaced by artificial joints made of metals or plastic, also with very small coefficients of friction



Friction as component

- The contact force between two objects can always be represented in terms of a normal force $\vec{n}$ perpendicular to surface of contact and a **friction force** $\vec{f}$ parallel to surface



Kinetic friction

- When an object is sliding over the surface, the friction force is called **kinetic friction** $\vec{f}_k$. Its magnitude f_k is approximately

$$f_k = \mu_k n$$

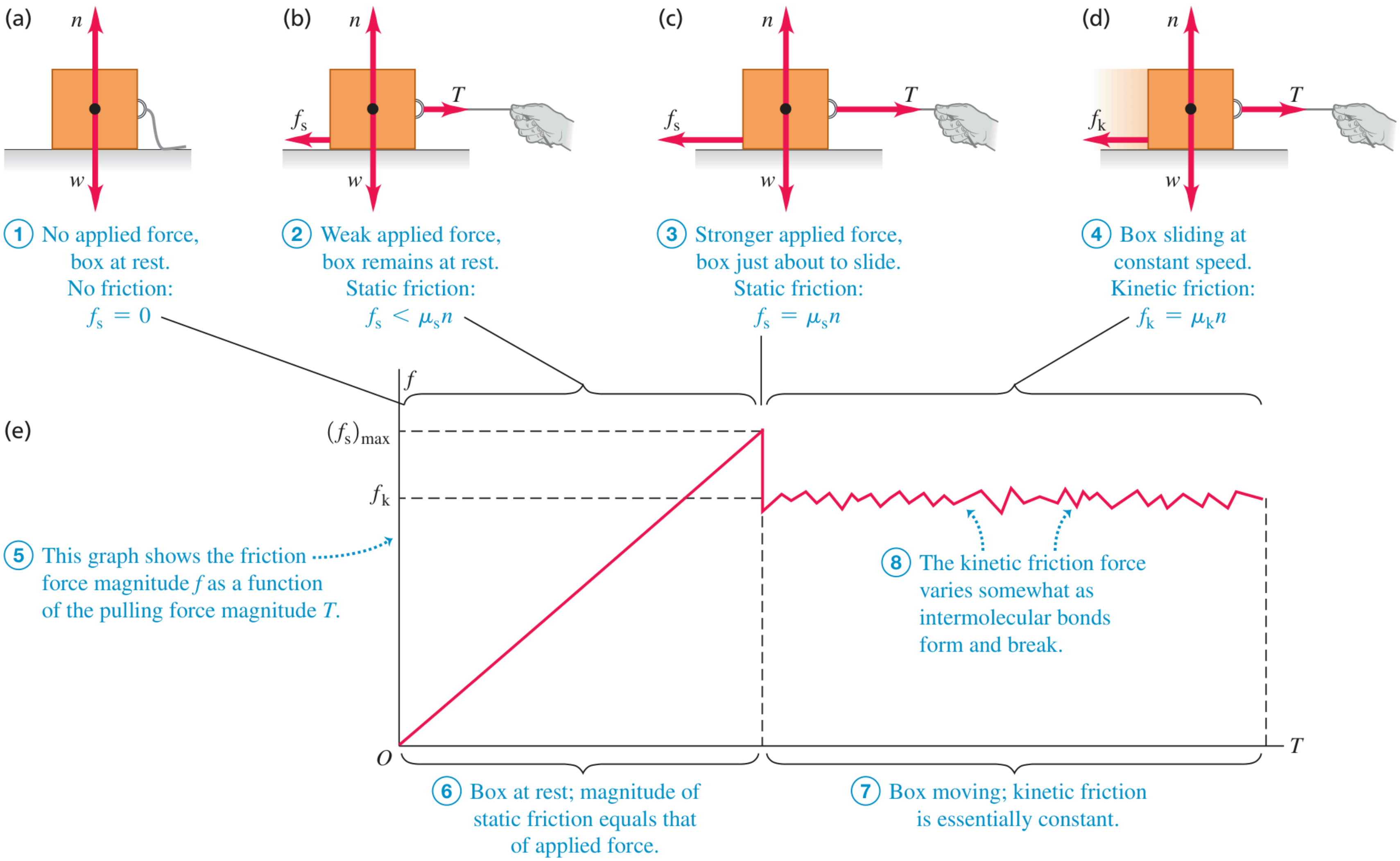
  where μ_k is the coefficient of kinetic friction
  - The more slippery the surface, the smaller this coefficient
- Friction can also depend on speed of object relative to surface
  - For now, we'll ignore this effect and assume that μ_k and f_k are independent of speed

Static friction

- When an object is not moving relative to a surface, the friction force is called **static friction** $\vec{f}_s$
  - Its magnitude, which may be anything from zero to some maximum value depending on situation, is approximately

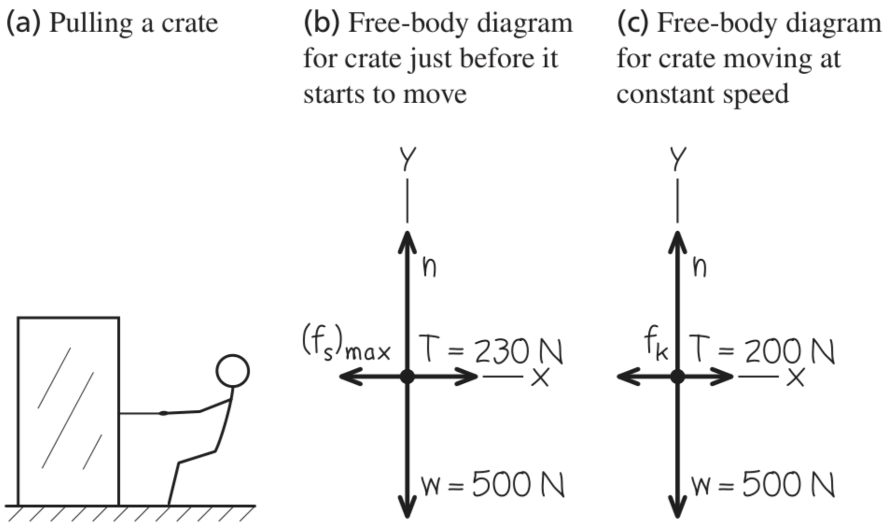
$$f_s \leq f_{s,\,\max} = \mu_s n$$

    where μ_s is the coefficient of static friction
  - Usually $\mu_s > \mu_k$ for a given pair of surfaces in contact



(a)
n
w
1 No applied force, box at rest. No friction: $f_s = 0$
(b)
n
T
f_s
w
2 Weak applied force, box remains at rest. Static friction: $f_s < \mu_s n$
(c)
n
T
f_s
w
3 Stronger applied force, box just about to slide. Static friction: $f_s = \mu_s n$
(d)
n
T
f_k
w
4 Box sliding at constant speed. Kinetic friction: $f_k = \mu_k n$
(e)
f
$(f_s)_{max}$
f_k
O
T
5 This graph shows the friction force magnitude f as a function of the pulling force magnitude T.
8 The kinetic friction force varies somewhat as intermolecular bonds form and break.
6 Box at rest; magnitude of static friction equals that of applied force.
7 Box moving; kinetic friction is essentially constant.

Example. You want to move a 500 N crate across a level floor. To start the crate moving, you have to pull with a 230 N horizontal force. Once the crate starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?



- Just before crate starts to move, we have via first law

$$\sum F_x = T + (-f_{s,\,\mathrm{max}}) = 0 \qquad \text{so} \qquad f_{s,\,\mathrm{max}} = T = 230\ \mathrm{N}$$

$$\sum F_y = n + (-w) = 0 \qquad \text{so} \qquad n = w = 500\ \mathrm{N}$$

- Solving for μ_s, we get

$$\mu_s = \frac{f_{s,\,\max}}{n} = \frac{230\ \text{N}}{500\ \text{N}} = 0.46$$

- After crate starts to move (constant $\vec{v}$), we have via first law

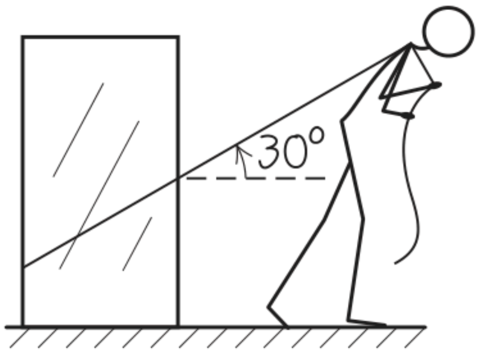
$$\sum F_x = T + (-f_k) = 0 \qquad \text{so} \qquad f_k = T = 200\ \text{N}$$

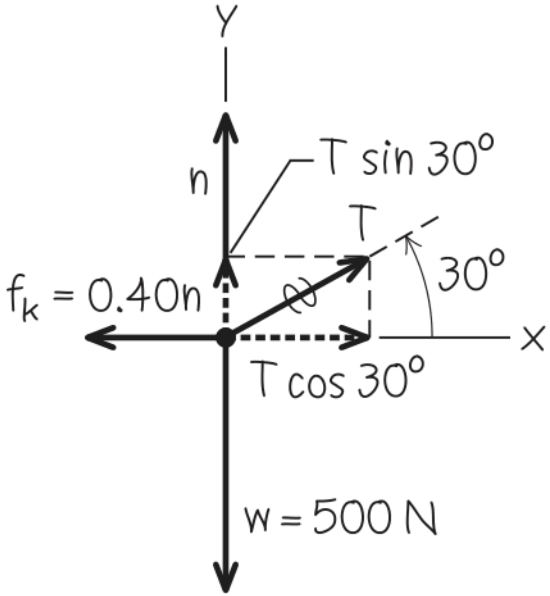
$$\sum F_y = n + (-w) = 0 \qquad \text{so} \qquad n = w = 500\ \text{N}$$

- Computing for μ_k, we find

$$\mu_k = \frac{f_k}{n} = \frac{200\ \text{N}}{500\ \text{N}} = 0.40$$

Example. In the previous example, suppose you move the crate by pulling upward on the rope at an angle of 30° above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that $\mu_k = 0.40$.





- The crate is in equilibrium because its velocity is constant so first law applies

- Kinetic friction force f_k is still equal to $\mu_k n$, but now the normal force n is not equal in magnitude to the crate's weight
  - The force exerted by the rope has a vertical component that tends to lift the crate off the floor
  - This reduces n and so reduces f_k
- The first law and the equation for kinetic friction give us

$$\sum F_x = T\cos 30° + (-f_k) = 0 \qquad \Longrightarrow T\cos 30° = \mu_k n,$$

$$\sum F_y = T\sin 30° + (-w) = 0 \qquad \Longrightarrow n = w - T\sin 30°$$

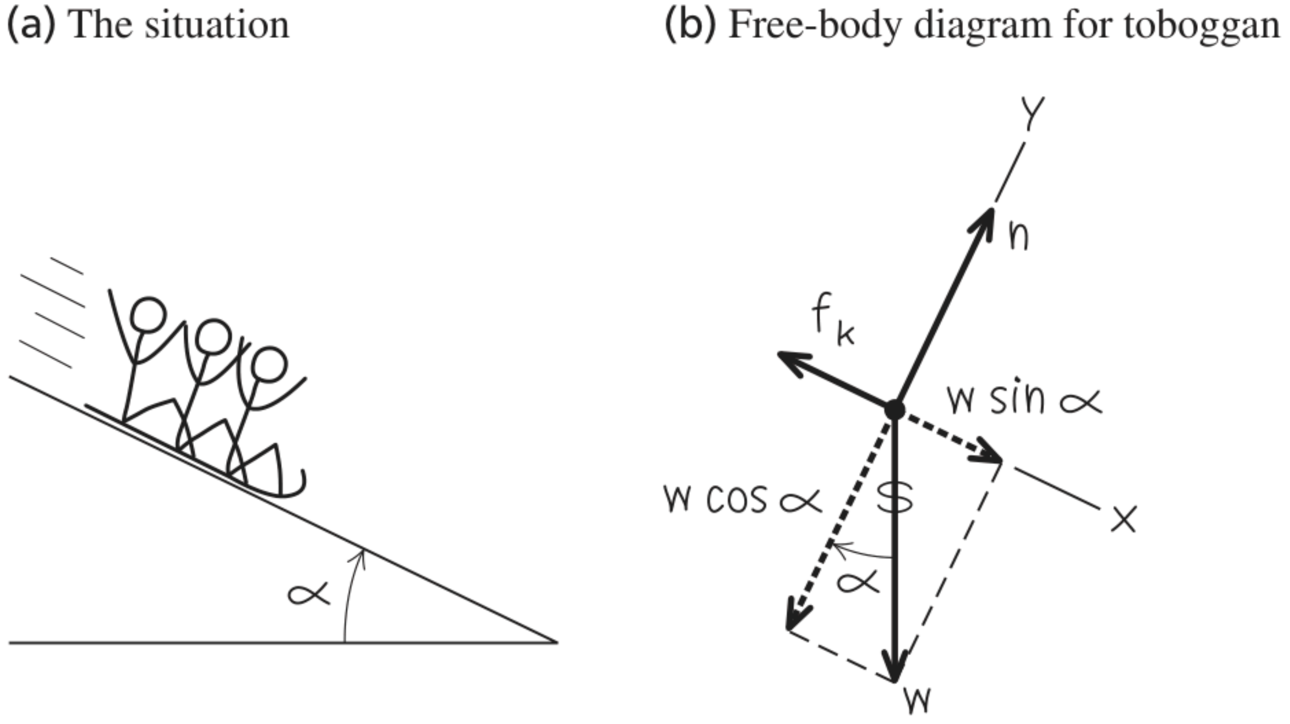
- Solving the two equations for two unknowns T and n by substituting the second result into the first, we get

$$T\cos 30° = \mu_k(w - T\sin 30)$$
$$\implies T = \frac{\mu_k w}{\cos 30° + \mu_k \sin 30°} = 188\ \text{N}$$

- Finally, substituting this into either of the original equation to obtain n, we get

$$n = w - T\sin 30° = (500\ \text{N}) - (188\ \text{N})\sin 30° = 406\ \text{N}$$

Example. Let's go back to the toboggan we studied in the previous example. The wax has worn off, so there is now nonzero coefficient of kinetic friction μ_k. The slope has just the right angle to make the toboggan slide with constant velocity. Find this angle in terms of w and m_k.



- The toboggan is in equilibrium because its velocity is constant, so we use the first law as in

$$\sum F_x = w \sin\alpha + (-f_k) = w \sin\alpha - \mu_k n = 0,$$

$$\sum F_y = n + (-w \cos\alpha) = 0$$

- Rearranging we get

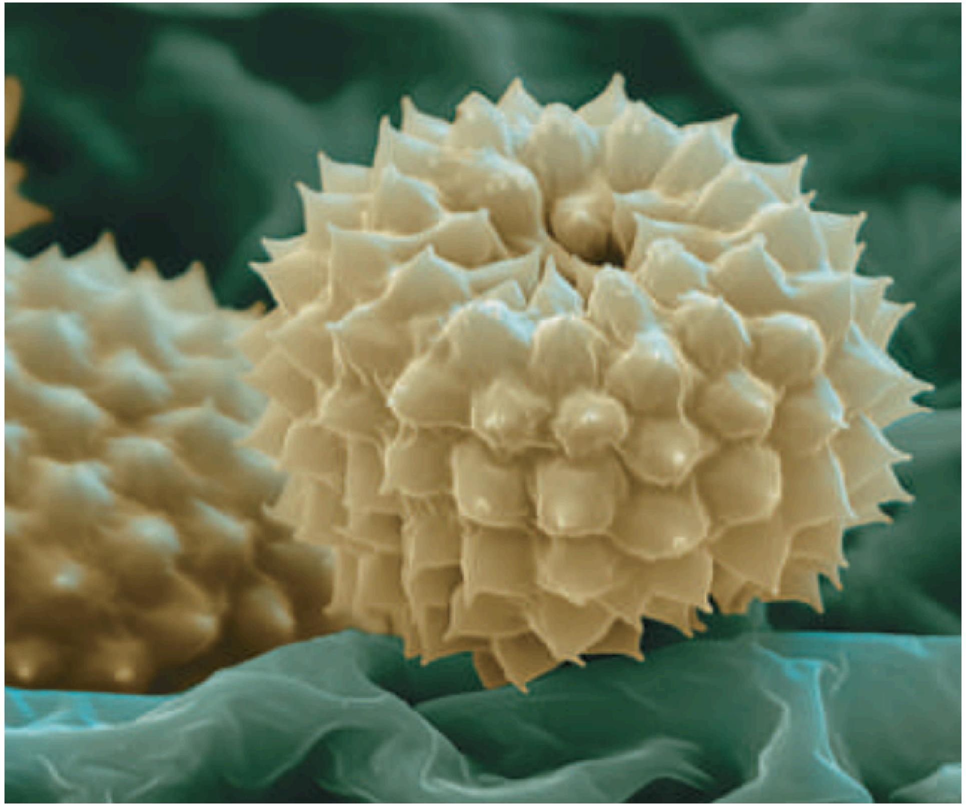
$$\mu_k n = w \sin\alpha, \qquad n = w \cos\alpha$$

- As in previous toboggan example, $n \neq w$. We eliminate n by dividing the first equation by the second giving us

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} \qquad \Longrightarrow \alpha = \arctan \mu_k$$

Fluid resistance

- The force that a fluid (a gas or liquid) exerts on an object moving through it is called **force of fluid resistance** $\boldsymbol{f}$
  - eg. ⟵🚗 🪟 🤳
- The moving object exerts force on fluid to push it out of way. By third law, fluid pushes back on the object



Fluid resistance

- For small objects moving at low speed v, its magnitude is

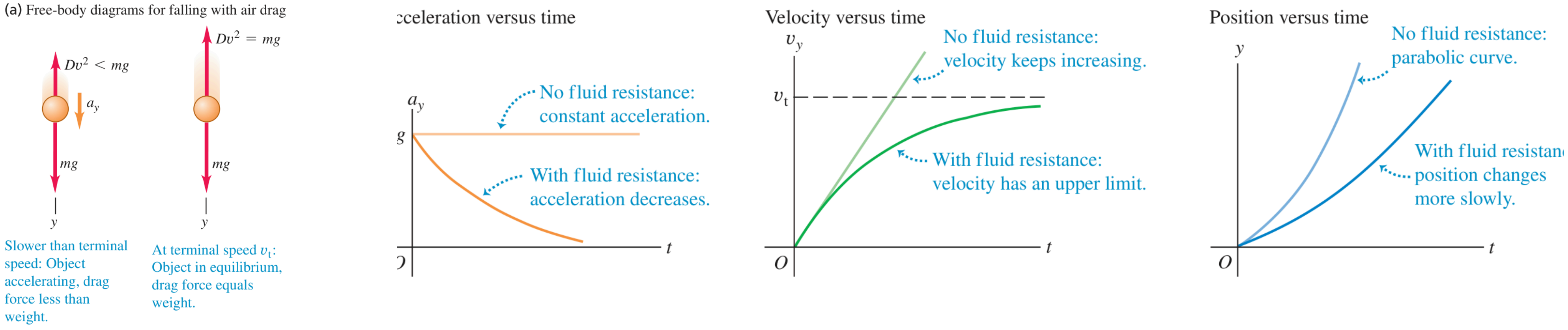
$$f = kv$$

  - Alternatively, using Stokes' law, $f = (6\pi\eta r)v$, where r is the radius of object and η is fluid viscosity. Above k has unit kg/s
- For larger objects moving thru air at speed of a tossed tennis ball or faster, resisting force is called **air drag**, or simply **drag**:

$$f = Dv^2$$

- Alternatively, $f = \left(\frac{1}{2}\rho AC\right)v^2$, where C is drag coefficient, A is area of the object facing the fluid, and ρ is the fluid density. Above D has unit kg/m

- As speed increases, resisting force also increases, until finally it is equal in magnitude to weight, wherein $a = 0$ and there is no further increase in speed. The final speed v_t is **terminal speed**



Example. For a human body falling through air in a spread-eagle position, the numerical value of the constant D is about 0.25 kg/m. Find the terminal speed for a 50 kg skydiver.

$$v_t = \sqrt{\frac{mg}{D}}$$

$$= \sqrt{\frac{(50\ \text{kg})(9.80\ \text{m/s}^2)}{0.25\ \text{kg/m}}}$$

$$= 44\ \text{m/s (about 160 km/h)}$$

Questions? 🤓

Elasticity 🪱

Elasticity

- Ideal rigid bodies don't bend or squash when forces act
  - But real things are elastic and deform to some extent
- The elastic properties of materials are very important
  - look at: bending of ✈️ wings, tendons in 🦴, suspensions in 🌁

Deformation

- The rigid body is a useful idealized model, but **deformations** like stretching, squeezing, and twisting of real objects when forces are applied are often too important to ignore
- **Hooke's law** states that in elastic deformations, **stress** (force per unit area) is proportional to **strain** (fractional deformation)
  - The proportionality constant is called the **elastic modulus**

$$\frac{\text{stress}}{\text{strain}} = \text{elastic modulus}$$

- Examples of stress
  - 🎸 strings under tensile stress, being stretched by forces acting at their ends
  - 🤿 under bulk stress, being squeezed from all sides by forces due to water pressure
  - 🎀 under shear stress, being deformed and eventually cut by forces exerted by the scissors

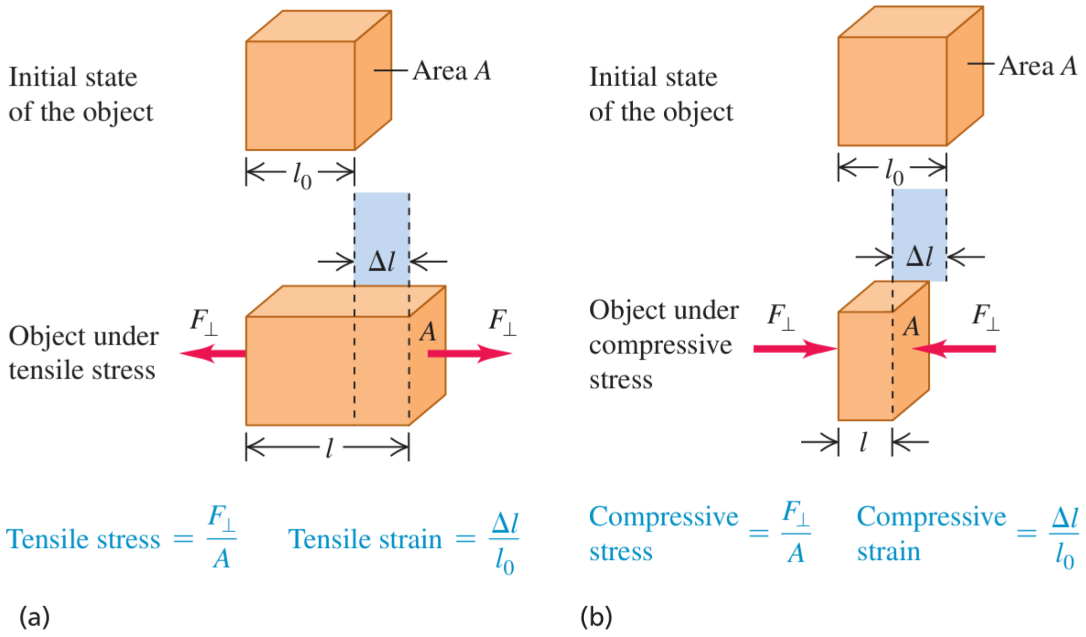
  

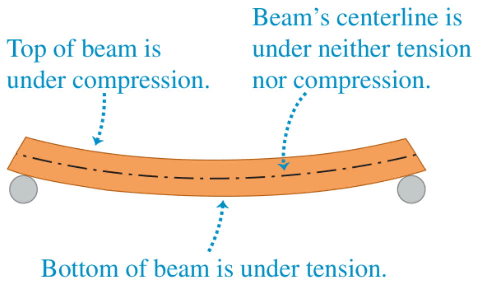
Tensile and compressive stress

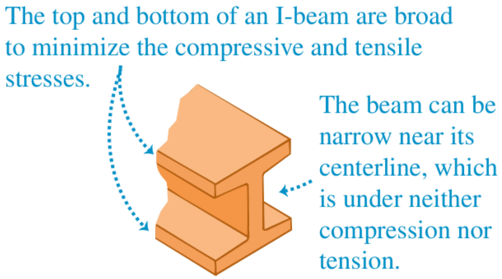
- An object subject to forces of equal magnitude $F_\perp$ but opposite directions at its ends, via a pull, is said to be in **tension**
- **Tensile stress** is tensile force per unit area $F_\perp/A$
  - It is a scalar quantity because $F_\perp$ is the magnitude of force
  - Its SI unit is pascal (Pa), where 1 Pa = 1 N/m^2
- **Tensile strain** is fractional change in length $\Delta l/l_0$
  - Here, strain is due to elongation Δl

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_\perp / A}{\Delta l / l_0} = \frac{F_\perp}{A} \frac{l_0}{\Delta l}$$

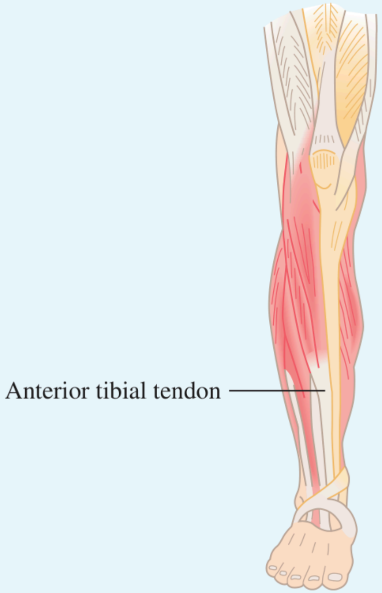
- The elastic modulus for tension is called **Young's modulus $\boldsymbol{Y}$**
- Whereas an object subject to forces on its ends by pushes rather than pulls is said to be in **compression**
    - **Compressive stress** and **compressive strain** are defined in the same way as above







BIO APPLICATION Young's Modulus of a Tendon The anterior tibial tendon connects your foot to the large muscle that runs along the side of your shinbone. (You can feel this tendon at the front of your ankle.) Measurements show that this tendon has a Young's modulus of 1.2×10^9 Pa, much less than for the metals listed in Table 11.1. Hence this tendon stretches substantially (up to 2.5% of its length) in response to the stresses experienced in walking and running.



- approximate elastic moduli

material	Young's Y ($\times 10^{10}$ Pa)	bulk modulus B ($\times 10^{10}$ Pa)	shear modulus S ($\times 10^{10}$ Pa)
aluminum	7.0	7.5	2.5
brass	9.0	6.0	3.5
copper	11	14	4.4
iron	21	16	7.7
lead	1.6	4.1	0.6
nickel	21	17	7.8
silicone rubber	0.001	0.2	0.0002
steel	20	16	7.5
tendon (typ.)	0.12	-	-

Example. A steel rod 2.0 m long has a cross-sectional area of 0.30 cm^2. It is hung by one end from a support, and a 550 kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation.

$$\text{tensile stress} = \frac{F_\perp}{A} = \frac{(550\ \text{kg})(9.8\ \text{m/s}^2)}{3.0 \times 10^{-5}\ \text{m}^2} = 1.8 \times 10^8\ \text{Pa}$$

$$\text{strain} = \frac{\Delta l}{l_0} = \frac{\text{stress}}{Y} = \frac{1.8 \times 10^8\ \text{Pa}}{20 \times 10^{10}\ \text{Pa}} = 9.0 \times 10^{-4}$$

$$\begin{aligned}\text{elongation} = \Delta l &= \text{strain} \times l_0 = \left(9.0 \times 10^{-4}\right)(2.0\ \text{m}) \\ &= 0.0018\ \text{m} = 1.8\ \text{mm}\end{aligned}$$

Bulk stress

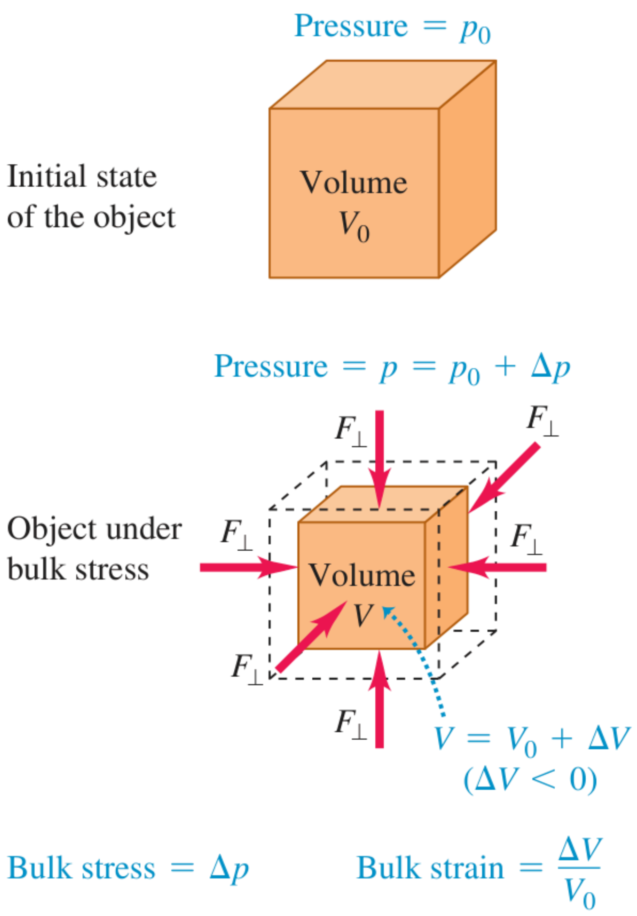
- Pressure in a fluid is force per unit area
  - Its SI unit is the same as stress, pascal (Pa)
  - Atmosphere (atm) is also commonly used, and is the approx. average pressure of earth's atmosphere at sea level:

$$1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$$

- **Bulk stress** is pressure change Δp, and **bulk strain** is fractional volume change $\Delta V / V_0$
- **Compressibility $\boldsymbol{k}$** is the reciprocal of bulk modulus: $k = 1/B$

- The elastic modulus for compression is called **bulk modulus** $\boldsymbol{B}$

$$p = \frac{F_\perp}{A}, \qquad B = \frac{\text{bulk stress}}{\text{bulk strain}} = \frac{\Delta p}{\Delta V / V_0}$$



BIO APPLICATION Bulk Stress on an Anglerfish The anglerfish (*Melanocetus johnsonii*) is found in oceans throughout the world at depths as great as 1000 m, where the pressure (that is, the bulk stress) is about 100 atmospheres. Anglerfish are able to withstand such stress because they have no internal air spaces, unlike fish found in the upper ocean, where pressures are lower. The largest anglerfish are about 12 cm (5 in.) long.

Example. A hydraulic press contains 0.25 m^3 (250 L) of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase $\Delta p = 1.6 \times 10^7$ Pa (about 160 atm). The bulk modulus of the oil is $B = 5.0 \times 10^9$ Pa (about 5.0×10^4 atm), and its compressibility is $k = 1/B = 20 \times 10^{-6}\ \text{atm}^{-1}$.

$$\Delta V = -\frac{V_0 \Delta p}{B} = -\frac{(0.25\ \text{m}^3)(1.6 \times 10^7\ \text{Pa})}{5.0 \times 10^9\ \text{Pa}}$$

$$= -8.0 \times 10^{-4}\ \text{m}^3 = -0.80\ \text{L}$$

$$\text{alternatively} : \Delta V = -kV_0\Delta p = -(20 \times 10^{-6}\ \text{atm}^{-1})(0.25\ \text{m}^3)(160\ \text{atm})$$

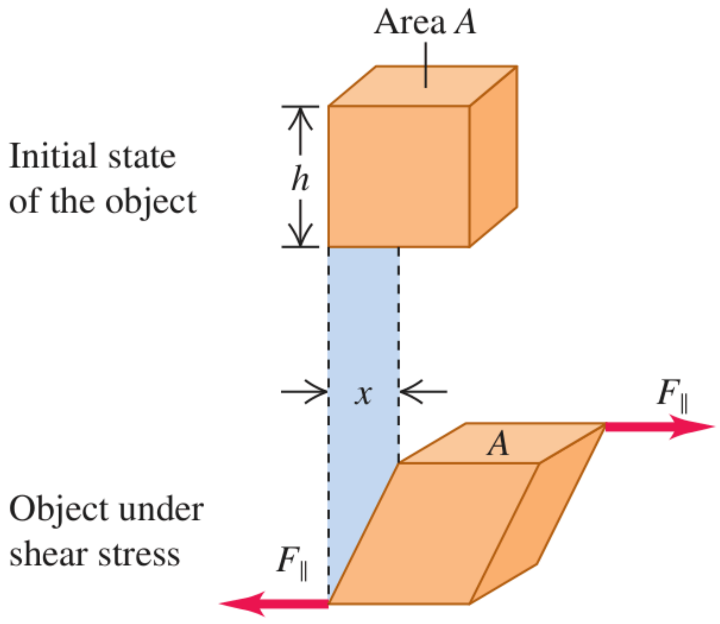
$$= -8.0 \times 10^-4\ \text{m}^3$$

Shear stress

- An object subject to forces that are tangent to opposite surface (as in forces act perpendicularly to the surfaces) is said to be under **shear** stress
- **Shear stress** is force per unit area $F_{\parallel}/A$ for a force applied tangent to a surface
- **Shear strain** is the displacement x of one side divided by the transverse dimension h
  - In real-life situations, x is typically much smaller than h

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A}\frac{h}{x}$$

- The elastic modulus for shear is called the **shear modulus** $\boldsymbol{S}$



$$\text{Shear stress} = \frac{F_{\parallel}}{A} \qquad \text{Shear strain} = \frac{x}{h}$$

Example. Suppose the object in the previous figure is the brass base plate of an outdoor sculpture that experiences shear forces in an eartquake. The plate is 0.80 m square and 0.50 cm thick. What is the force exerted on each of its edges if the resulting displacement x is 0.16 mm?

$$\text{shear strain} = \frac{x}{h} = \frac{1.6 \times 10^{-4}\ \text{m}}{0.80\ \text{m}} = 2.0 \times 10^{-4}$$

$$\text{shear stress} = \text{strain} \times S = 2.0 \times 10^{-4}(3.5 \times 10^{10}\ \text{Pa}) = 7.0 \times 10^{6}\ \text{Pa}$$

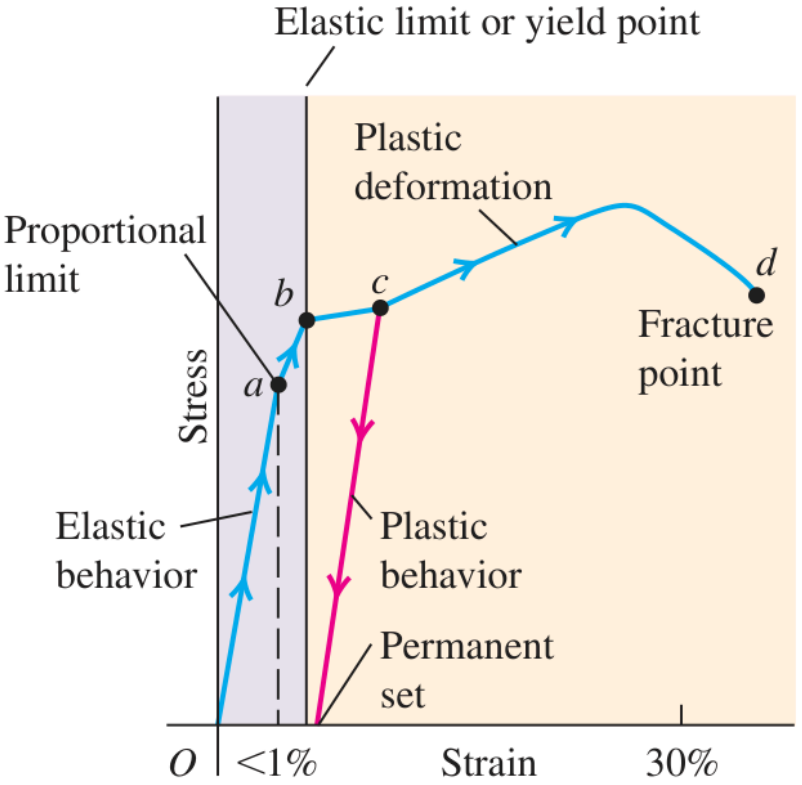
$$\text{finally}: F_{\|} = \text{stress} \times A = (7.0 \times 10^{6}\ \text{Pa})(0.80\ \text{m})(0.0050\ \text{m})$$

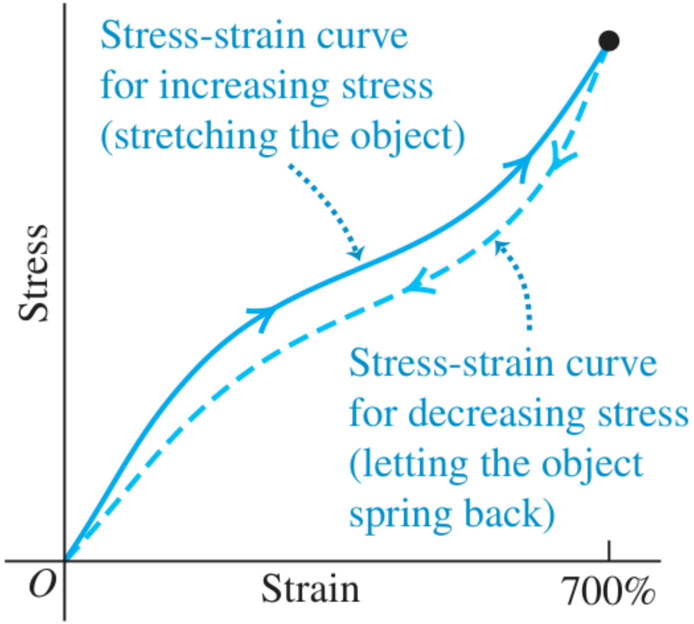
$$= 2.8 \times 10^{4}\ \text{N}$$

Limits of Hooke's law

- The **proportional limit** is the maximum stress for which stress and strain are proportional
  - Beyond the proportional limit, Hooke's law is not obeyed
  - But material is still elastic and can return to its original form
- The **elastic limit** is the stress beyond which irreversible deformation occurs
  - Material does not return to original form and has acquired a permanent set. This is where **plastic** behavior begins

- The **breaking stress**, or ultimate strength, is the stress at which the material breaks
  - Here, small additional stress produces a relatively large increase in strain, until fracture takes place
- Unlike uniform materials such as metals, stretchable biological materials like tendons and ligaments have no true plastic region
  - This is due to them being made of a collection of microscopic fibers, where they instead tear apart from each other
- **Elastic hysteresis** is when material follows different curves for increasing and decreasing stress, mainly due to internal friction


Elastic limit or yield point
Plastic deformation
Proportional limit
b
c
d
a
Fracture point
Stress
Elastic behavior
Plastic behavior
Permanent set
O
<1%
Strain
30%

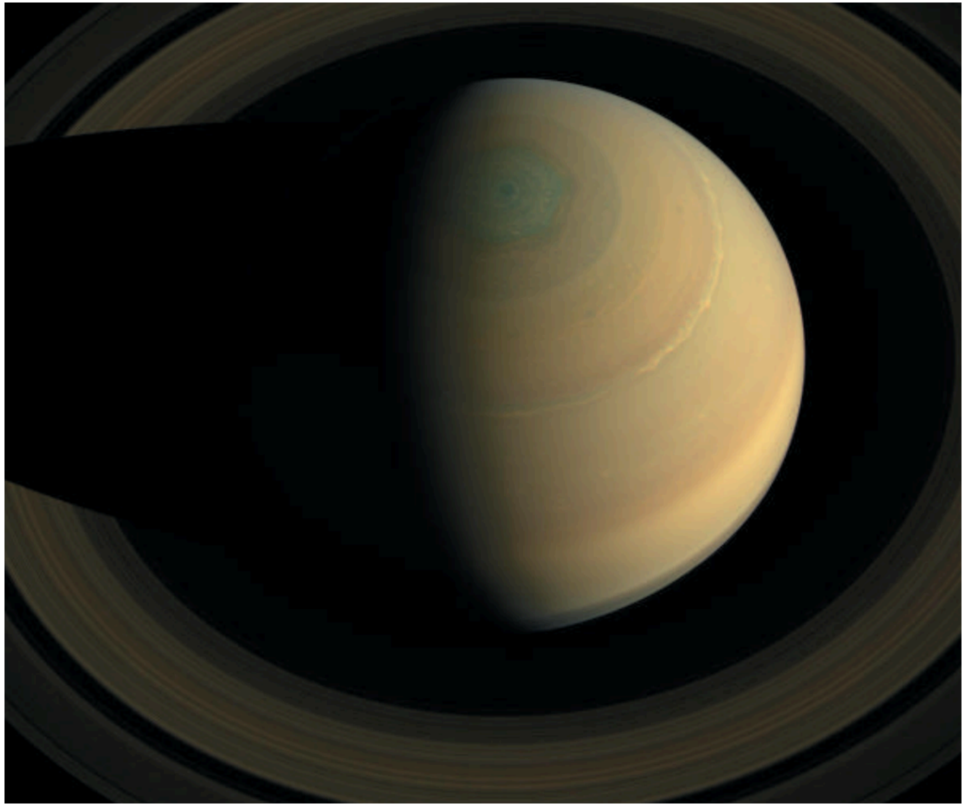

Stress-strain curve for increasing stress (stretching the object)
Stress
Stress-strain curve for decreasing stress (letting the object spring back)
O
Strain
700%

Questions? 🙄

Gravitation 🪐

Gravitation

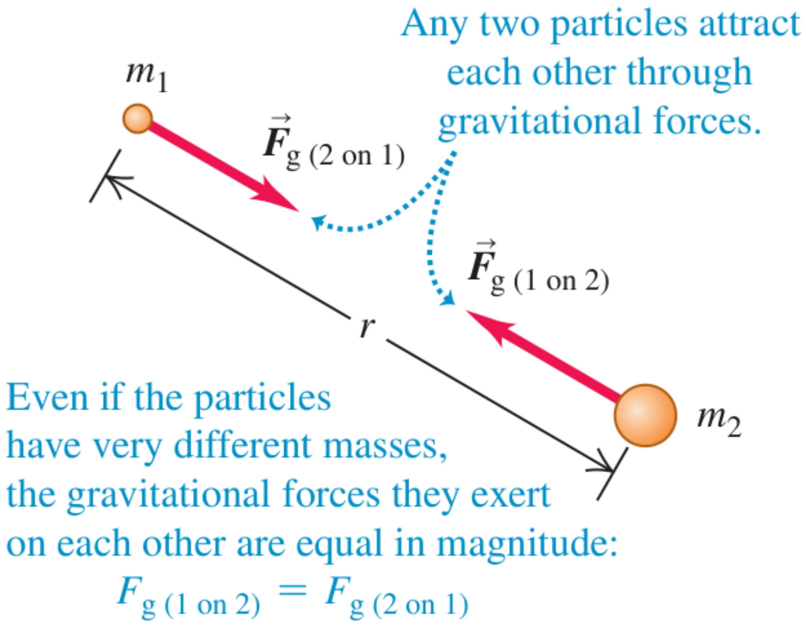
- Gravitation is one of the four classes of interactions found in nature (as prev. discussed)
- It answers questions about anything celestial
  - Why doesn't 🌙 fall earth?
  - Why doesn't earth fly off into space, it remains in orbit around sun instead?



Newton's law of gravitation

- Any two particles with masses m_1 and m_2, a distance r apart, attract each other with forces inversely proportional to r^2

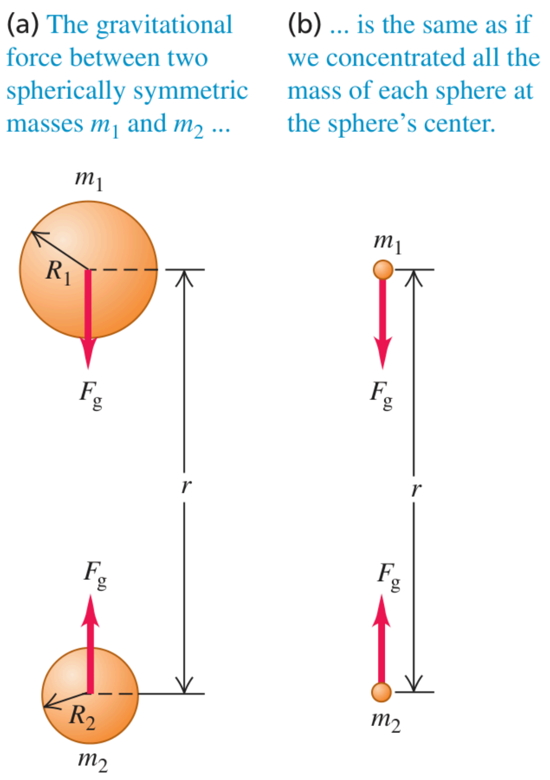
$$F_g = \frac{Gm_1m_2}{r^2}$$



Newton’s law of gravitation

- These forces form an action-reaction pair and obey Newton’s third law
- The accepted value of the gravitational constant G as of 2018 is

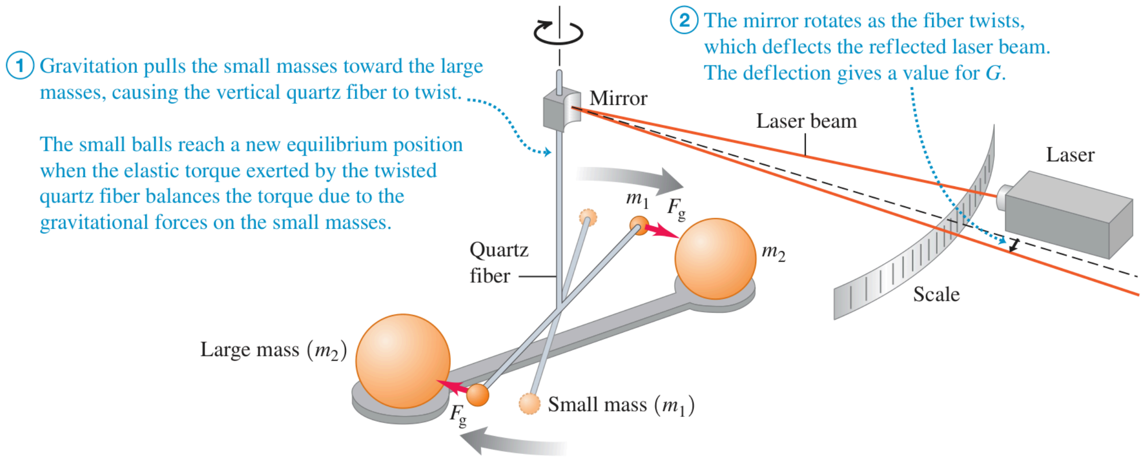
$$G = 6.67408 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$$



Don't confuse G with g

- Lowercase g is acceleration due to gravity, which relates weight w of an object to its mass m: $w = mg$
  - The value of g is different at different locations on the earth's surface and on the surfaces of other planets
- By contrast, capital G relates gravitational force between any two objects to their masses and distance between them
  - We call G a universal constant because it has same value for any two objects, no matter where in space they are located

Example. The mass m_1 of one of the small spheres of a Cavendish balance is 0.0100 kg, the mass m_2 of the nearest large sphere is 0.500 kg, and the center-to-center distance between them is 0.0500 m. Find the gravitational force F_g on each sphere due to the other.



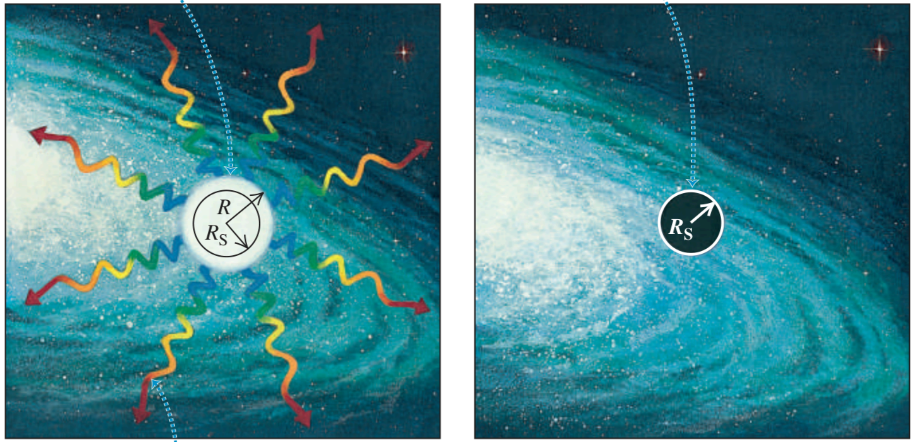
$$F_g = \frac{(6.67\times10^{-11}\ \mathrm{N\ m^2/kg^2})(0.0100\ \mathrm{kg})(0.500\ \mathrm{kg})}{(0.0500\ \mathrm{m})^2} = 1.33\times10^{-10}\ \mathrm{N}$$

Black holes

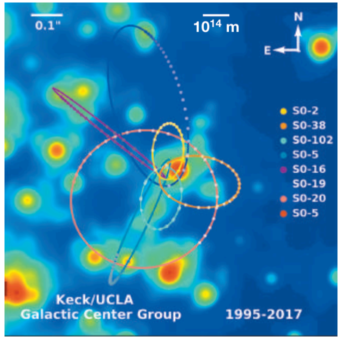
- If a nonrotating spherical mass distribution with total mass M has a radius less than its Schwarzschild radius $R_S = 2GM/c^2$, it is called a **black hole**
- The gravitational interaction prevents anything, including light, from escaping from within a sphere with radius R_S

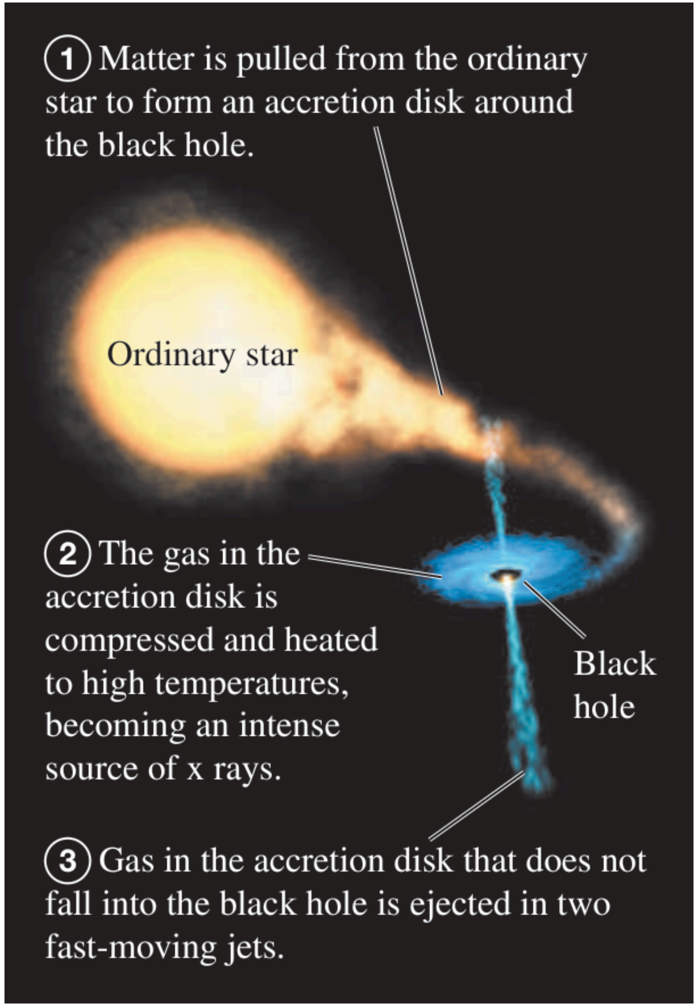
(a) When the radius R of an object is greater than the Schwarzschild radius R_S, light can escape from the surface of the object.

(b) If all the mass of the object lies inside radius R_S, the object is a black hole: No light can escape from it.



Gravity acting on the escaping light "red shifts" it to longer wavelengths.





Quiz time 🕐

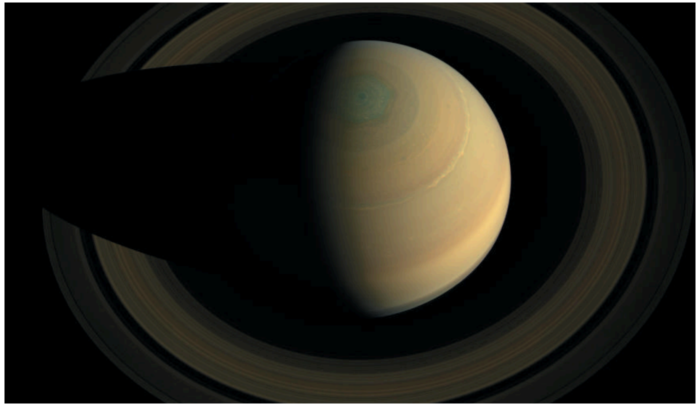
Hell nah, that steel post wasn't there 🅿️

While parking your car, you accidentally back into a steel post. You pull forward until the car no longer touches the post and then get out to inspect the damage. What does your rear bumper look like if the strain in the impact was

- (a) less than at the proportional limit
- (b) greater than at proportional lim but less than at yield point
- (c) greater than at yield point but less than at fracture point
- (d) greater than at the fracture point?

Where is it faster? 🪐

Choose one and tell me why.
The rings of Saturn are made of countless individual orbiting particles. Compared with a ring particle that orbits far from Saturn, does a ring particle close to Saturn orbit with...



- same speed and greater accel.
- faster speed and same accel.
- slower speed and same accel.
- faster speed, greater accel.