Forces and laws of motion, applied

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2025 W37[1]

[1] Phys 20.01 Mod 2. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously ⏪

Some preliminaries 📕

Applying Newton's laws ⚖️

Friction forces 🐌

Elasticity 🪱

Quiz time 🕐

Previously ⏪

Forces, laws of motion

First law: which one?

Example. In which case is there zero net external force on object?

- ✈️ flying due north at steady 120 m/s and at constant altitude
- 🚗 driving straight up hill with 3° slope at constant 90 km/h
- 🐦 circling at constant 20 km/h at constant height of 15 m
- 📦 with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s^2

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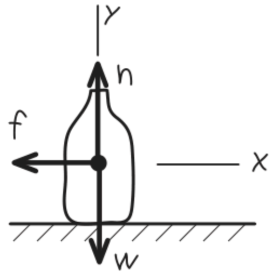
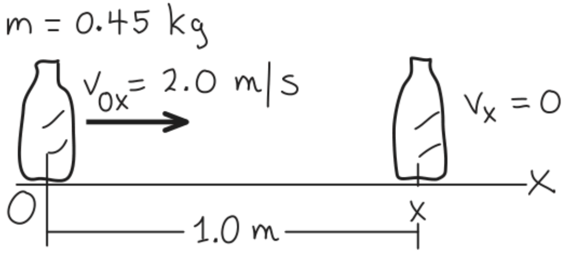
- Therefore answer is ✈️ 🚗 📦

Second law: ketchup later?

Example. A waitress shoves a ketchup bottle with mass 0.45 kg to her right along a smooth, level lunch counter. The bottle leaves her hand moving at 2.0 m/s, then slows down as it slides because of a constant horizontal friction force exerted on it by the countertop. It slides for 1.0 m before coming to rest. What are the magnitude and direction of the friction force acting on the bottle?

Second law: ketchup later?





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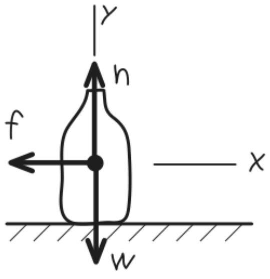

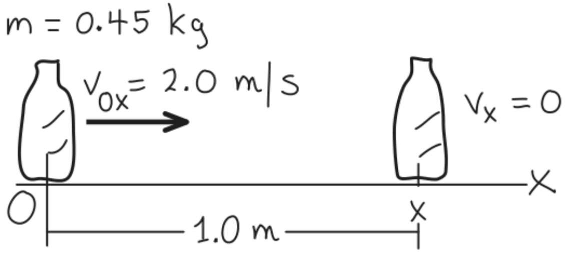

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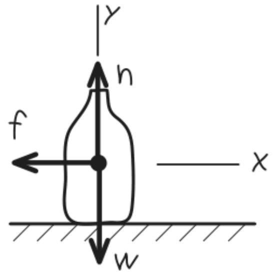

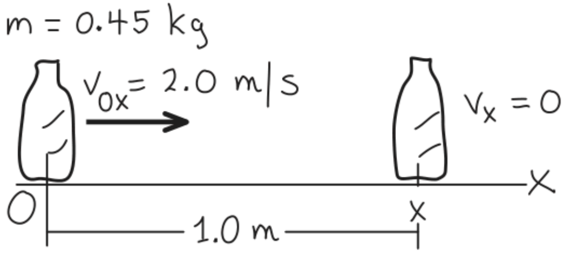

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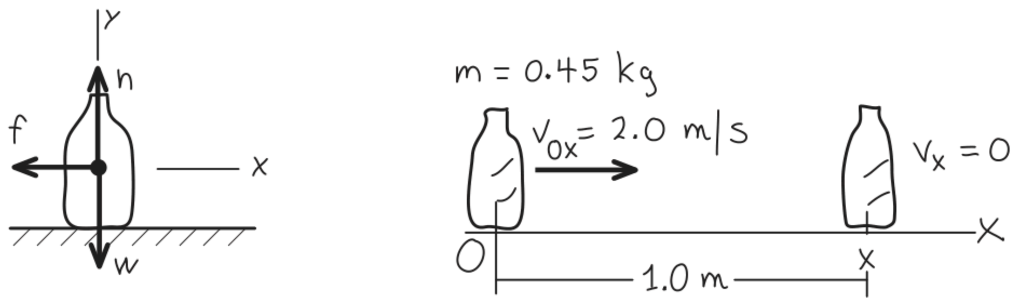

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$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(0\ \mathrm{m/s})^2 - (2.0\ \mathrm{m/s})^2}{2(1.0\ \mathrm{m} - 0\ \mathrm{m})} = -2.0\mathrm{m/s}^2$$

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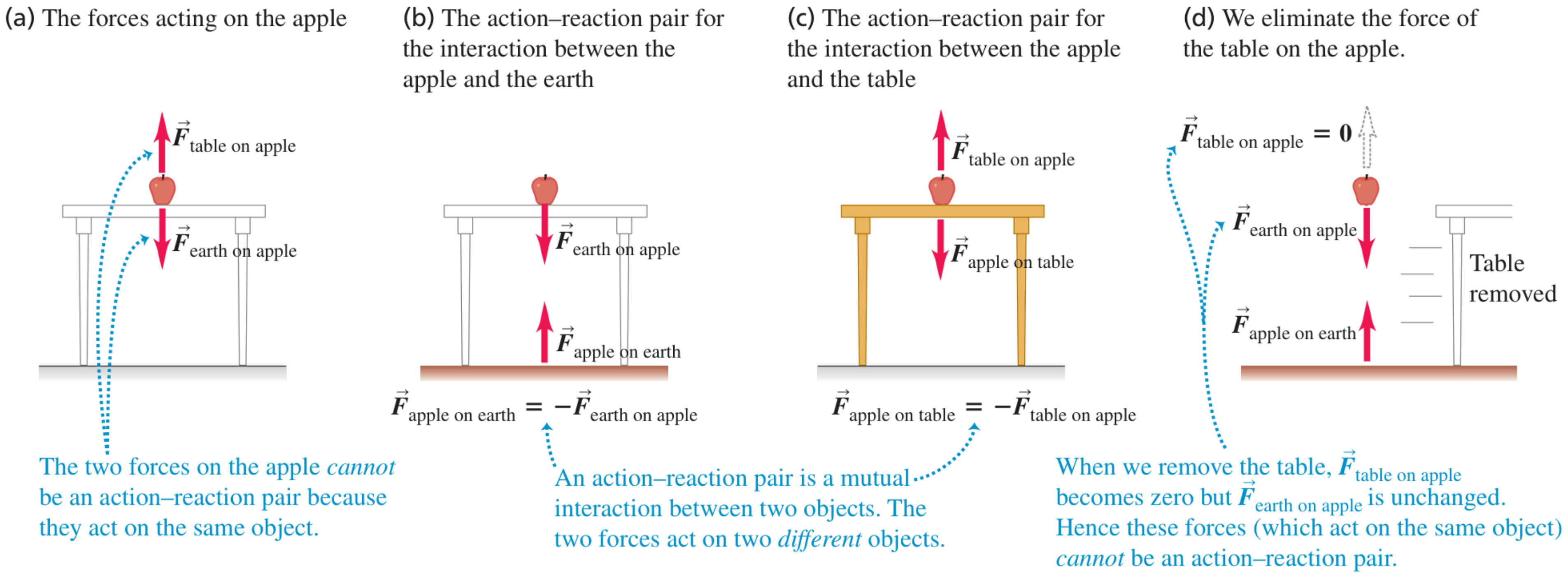
- The negative sign shows that net external force on the bottle is towards left. The magnitude of friction force is then $f = 0.90$ N

Third law: awe dropping?

Example. An apple sits at rest on a table, in equilibrium. What forces act on the apple? What is the reaction force to each of the forces acting on the apple? What are the action-reaction pairs?



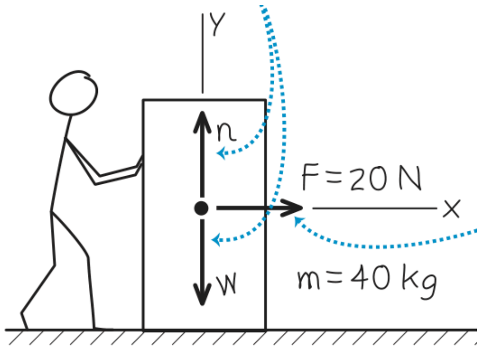
- The two forces in the action-reaction pair always act on two different objects

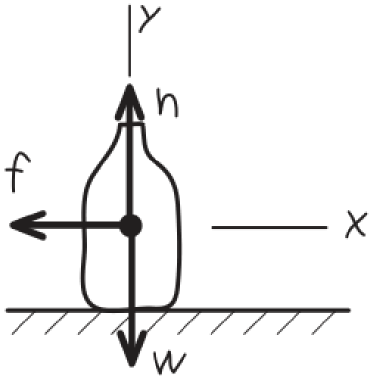
Questions? 🤓

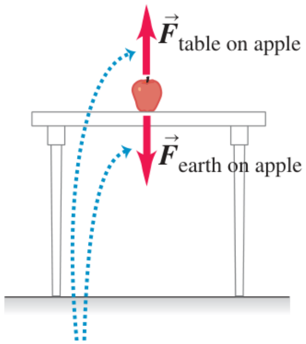
Some preliminaries 📕

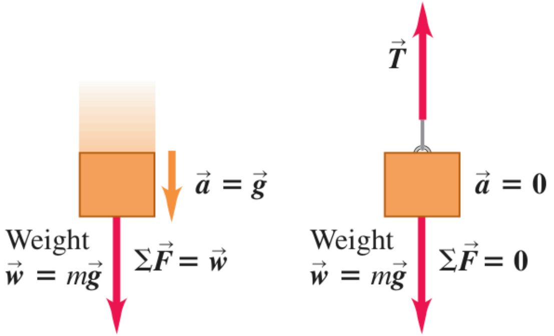
Free-body diagram

- A **free-body diagram** shows the object we choose to analyze by itself, "free" of its surroundings, with vectors drawn to show magnitudes and directions of all forces that act on the object
  - Here, body is another word for object

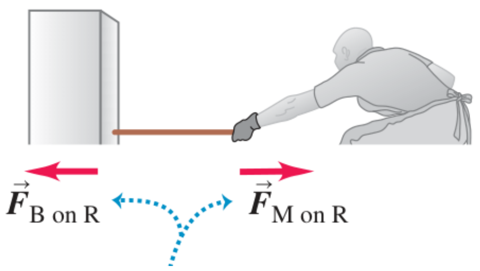


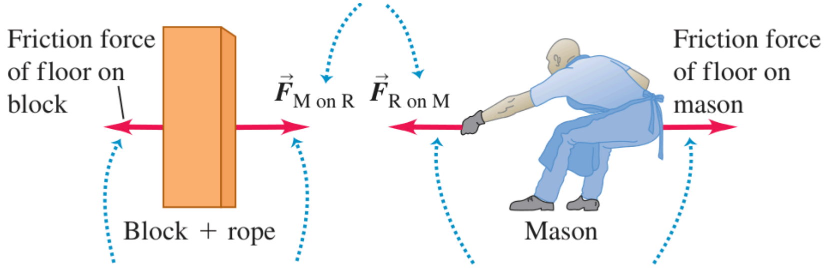




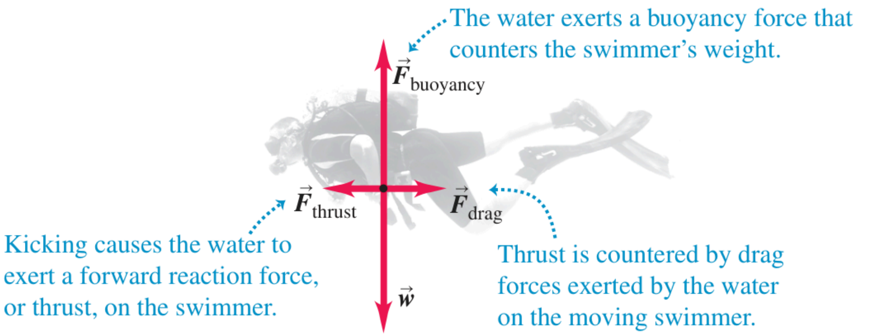


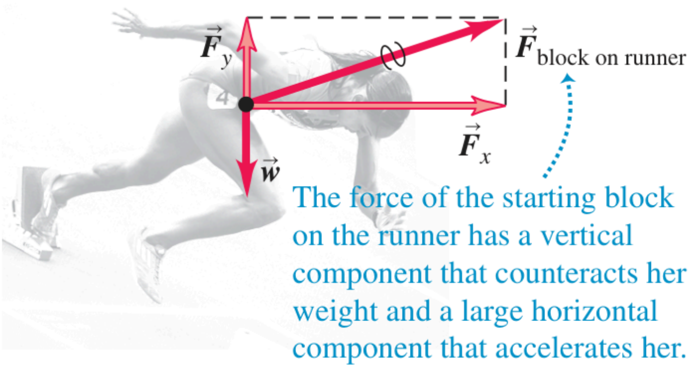
- When a problem involves more than one object, we take the problem apart and draw a separate free-body diagram for each

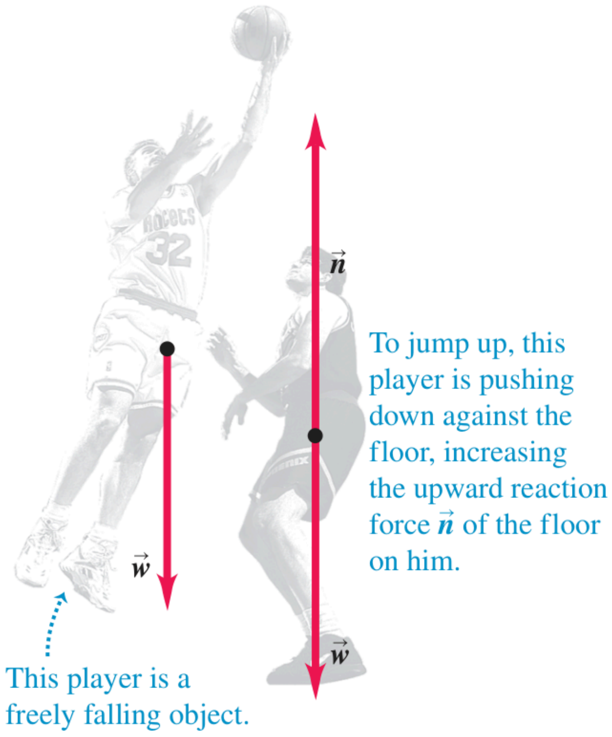




- Some real-life situations and corresponding free-body diagrams




$\vec{F}_y$
$\vec{F}_{\text{block on runner}}$
θ
$\vec{F}_x$
$\vec{w}$
The force of the starting block on the runner has a vertical component that counteracts her weight and a large horizontal component that accelerates her.


$\vec{n}$
$\vec{w}$
$\vec{w}$
To jump up, this player is pushing down against the floor, increasing the upward reaction force $\vec{n}$ of the floor on him.
This player is a freely falling object.

Inertial frame of reference

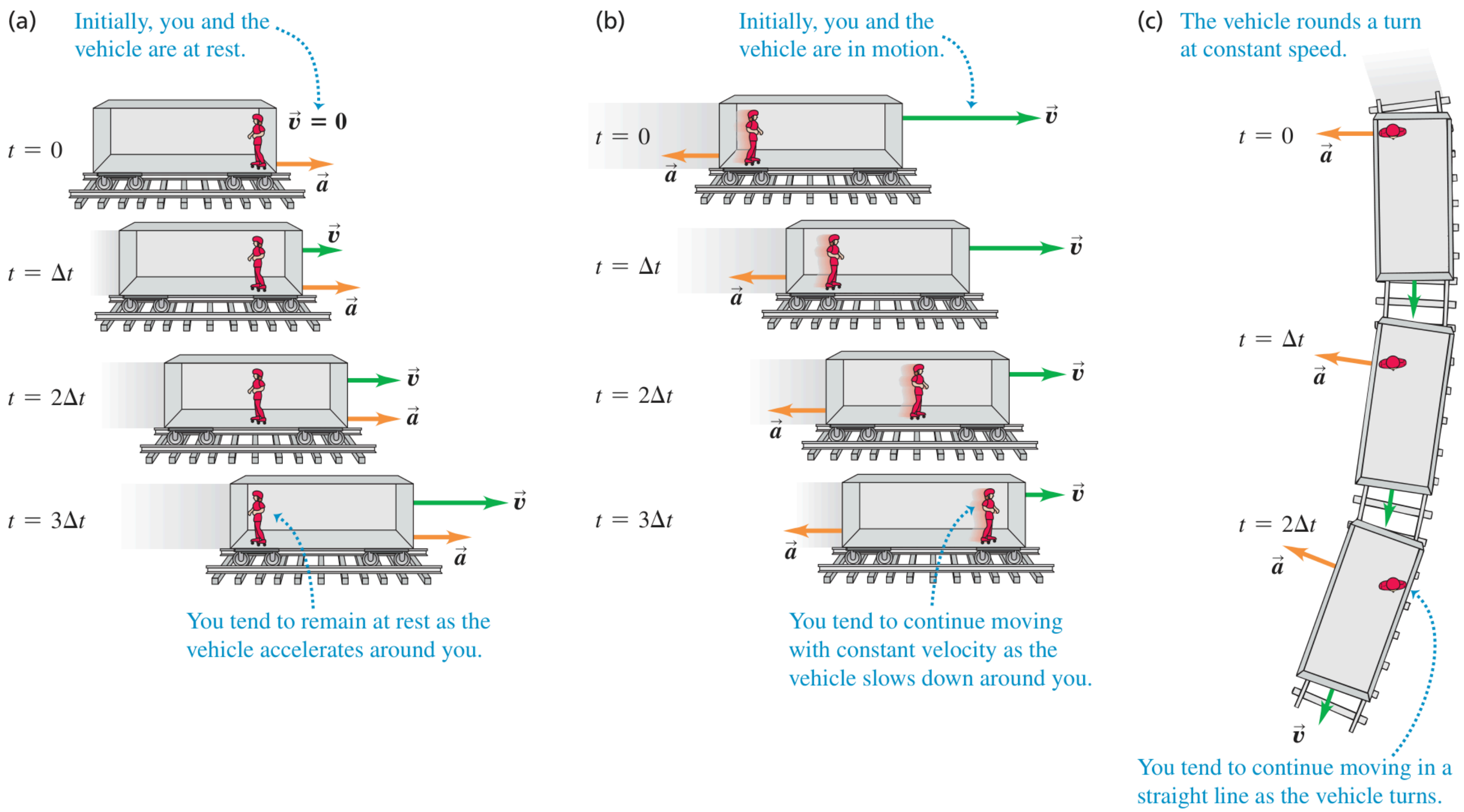
- Recall the concept of **reference frame** in kinematics
  - eg. 🧑 🛼 🚃 $\rightarrow \longrightarrow$ (accelerating)
  - Relative to people in the train, you start moving backwards from rest as train gains speed. Relative to people outside, you remain at rest. Newton's first law applies differently?
  - Relative to train frame, the first law says you don't move with constant velocity $\rightarrow$ you accelerate $\rightarrow$ net external force $\neq 0$
  - Relative to earth frame, the first law says you remain at rest $\rightarrow$ you don't accelerate $\rightarrow$ net external force on you $= 0$

- Here, there's a more valid or suitable frame for Newton's laws

- That'd be **inertial frame of reference**, which is a frame that is not accelerating or rotating relative to another frame
  - Basically, it is where first law holds true. For practicality, a frame fixed to earth can be approximated as an inertial frame, though earth's rotation and orbit mean it isn't strictly inertial

- There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws
  - If one frame is inertial, then every other frame moving relative to it with constant velocity is also inertial

- eg. we pick inertial (earth) over non-inertial (accelerating train):



Notes on units and magnitudes

- In British system, unit of force is pound/pound-force (lb/lbf), unit of mass is slug, unit of distance is foot (ft), thus 1 lb = 1 slug ft/s^2
  - Official definition of pound is 1 lb = 4.448221615260 N

[1]

[1]Despite its name, the British unit of mass has nothing to do with the type of slug shown here. A common garden slug has a mass of about 15 grams, or about 10^{-3} slug.

Notes on units and magnitudes

- What we have been using is SI or metric system (m, kg, s; N)
- In cgs metric system, unit of mass is gram (g), equal to 10^{-3} kg, and unit of distance is centimeter (cm), equal to 10^{-2} m
  - Unit of force is dyne (dyn): 1 dyn = 1 g cm/s^2 = 10^{-5} N
- Handy to know!
  - A pound is about 4.4 N and a newton is about 0.22 pound
  - An object with a mass of 1 kg has a weight of about 2.2 lb at earth's surface

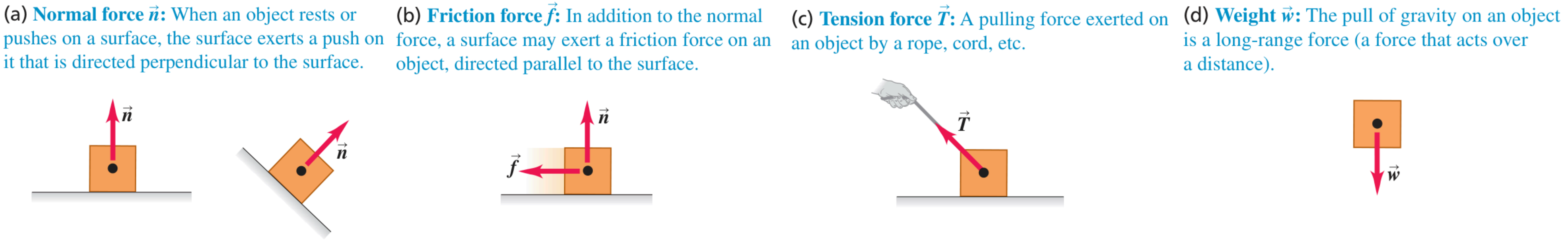
Typical force magnitudes in newtons (N)		
☀️	Sun's gravitational force on the earth	3.5×10^{22}
🐋	Weight of a large blue whale	1.9×10^{6}
🚂	Maximum pulling force of a locomotive	8.9×10^{5}
🏉	Weight of a 250 lb linebacker	1.1×10^{3}
🍎	Weight of a medium apple	1
🪰	Weight of the smallest insect eggs	2×10^{-6}
⚡	Electric attraction bet. p^+ and e^- in H atom	8.2×10^{-8}
🦠	Weight of a very small bacterium	1×10^{-18}
⚛️	Weight of a hydrogen atom	1.6×10^{-26}
🪐	Gravitational attraction bet. p^+, e^- in H atom	3.6×10^{-47}

Questions? 😦

Applying Newton's laws ⚖️

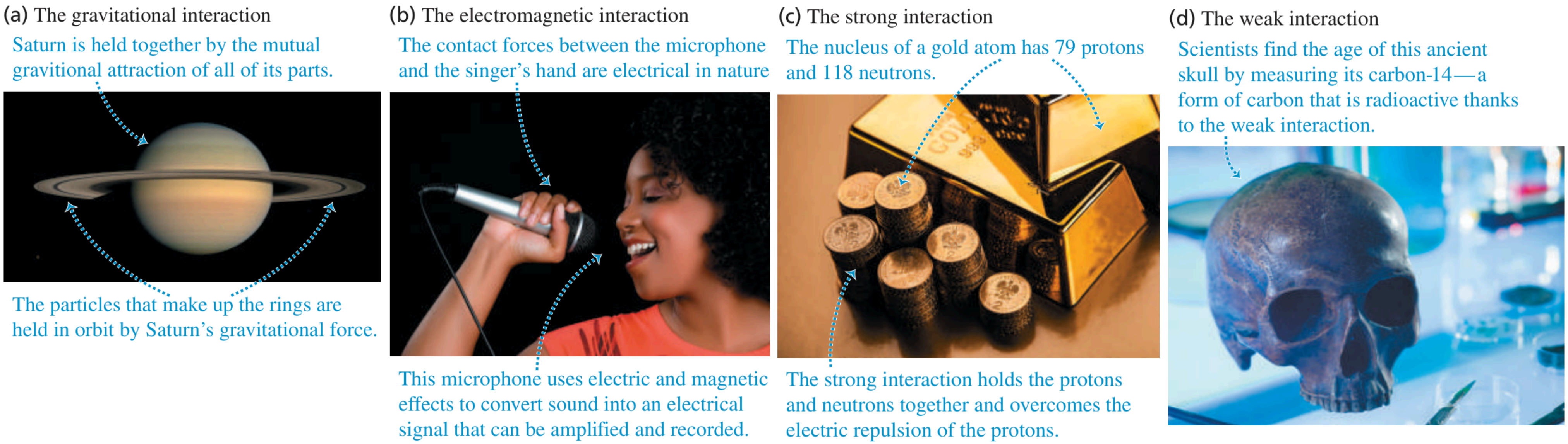
Common types of forces

- **Normal force $\vec{n}$** is exerted on an object by any surface with which it is in contact. Normal means that the force always acts perpendicular to the surface of contact, no matter what angle
  - Generally, $\vec{n} = -\vec{w} = -m\vec{g}$
- **Friction force $\vec{f}$** exerted on an object by a surface acts parallel to the surface, in the direction that opposes sliding
- The pulling force exerted by a stretched rope or cord on an object to which it's attached is **tension force $\vec{T}$**
  - $\vec{T} = -\vec{w}$ if object of interest is hanging straight down



- The earth pulls a dropped object toward it even though there is no direct contact between object and earth. This gravitational force that earth exerts on object is the object's **weight** $\vec{w}$
  - On an incline with angle θ, weight $\vec{w}$ can break into parts that act perpendicular ($\vec{w}_\perp$) and parallel ($\vec{w}_\parallel$) to the surface:
    $\vec{w}_\parallel = w \sin\theta = mg\sin\theta, \qquad \vec{w}_\perp = w\cos\theta = mg\cos\theta$

- Forces that are not really giving "daily life" vibes, but are still common, are the **fundamental forces** of nature: gravitational, electromagnetic, strong, and weak interactions



Quick tips

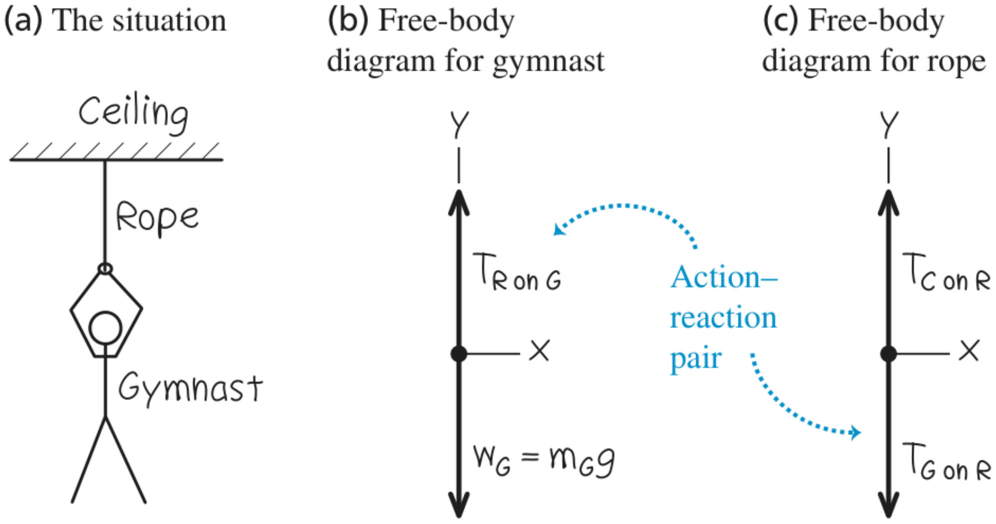
- Newton's first and second laws apply to a specific object
  - Decide which object you are referring when using these laws
- Only forces acting on the object matter
  - $\sum \vec{F}$ includes all forces that act on the object in question
  - Don't confuse forces acting on an object with the forces exerted by that object on some other object
- Free-body diagrams are essential to help identify relevant forces
  - Show the force vectors that act on the chosen object "free" of its surroundings

Using the first law: objects in equilibrium 🅰️

$$\sum \vec{F} = \vec{0}, \qquad \sum F_x = 0, \quad \sum F_y = 0$$

- When an object is in equilibrium in an inertial frame of reference, that is either at rest or moving with constant velocity, the vector sum of forces acting on it must be zero (first law)
  - eg. hanging lamp, kitchen table, ✈️ ➡️ at constant $\vec{v}$
- Third law is also frequently needed in equilibrium problems
  - The 2 forces in action-reaction pair never act on same object

Example. A gymnast with mass $m_G = 50.0$ kg suspends herself from the lower end of a hanging rope of negligible mass. The upper end of the rope is attached to the gymnasium ceiling. (a) What is the gymnast's weight? (b) What force (magnitude and direction) does rope exert on her? (c) What is the tension at the top of rope?



- (a) The magnitude of gymnast's weight is her mass times acceleration due to gravity:

$$w_G = m_G g = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

- (b) The gravitational force on gymnast (her weight) points in $-y$-direction so its y-component is $-w_G$. The upward force of rope on gymnast has unknown magnitude $T_{R\text{ on }G}$ and $+y$-component. Using first law,

$$\text{gymnast:} \qquad \sum F_y = T_{R\text{ on }G} + (-w_G) = 0 \qquad \text{so}$$

$$T_{R\text{ on }G} = w_G = 490\ \text{N}$$

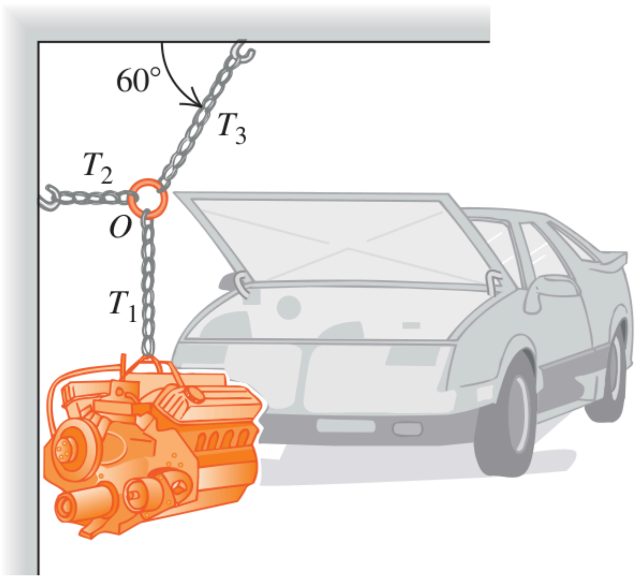
  - The rope pulls upward on gymnast with force $T_{R\text{ on }G}$ of magnitude 490 N

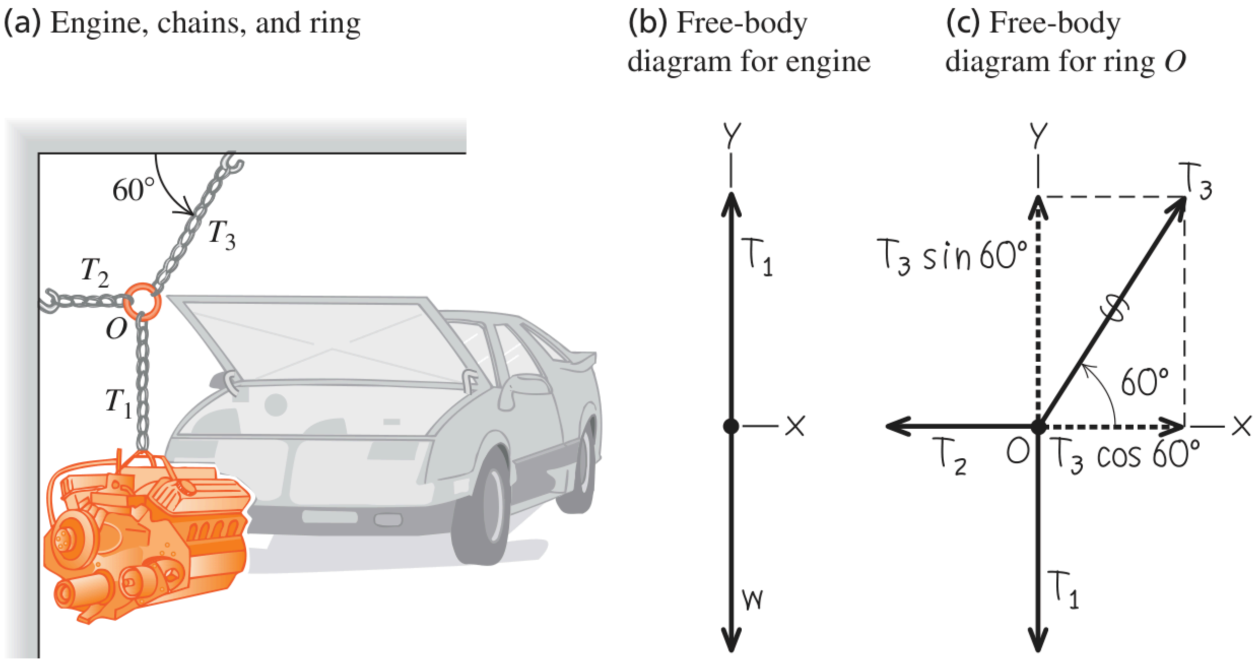
- (c) Assume rope is weightless, so only forces on it are those exerted by ceiling (upward tension $T_{C \text{ on } R}$) and by gymnast (downward $T_{G \text{ on } R} = 490$ N). From first law, net vertical force on rope in equilibrium must be zero, so

$$\text{rope:} \qquad \sum F_y = T_{C \text{ on } R} + (-T_{G \text{ on } R}) = 0 \qquad \text{so}$$
$$T_{C \text{ on } R} = T_{G \text{ on } R} = 490 \text{ N}$$

- Key concept: The sum of all the external forces on an object in equilibrium is zero. The tension has the same value at either end of a rope or string of negligible mass

Example. In figure, a car engine with weight w hangs from a chain that is linked at ring O to two other chains, one fastened to the ceiling and the other to the wall. Find expressions for the tension in each of three chains in terms of w. The weights of ring and chains are negligible compared with the weight of the engine.





- Forces acting on engine are along y-axis only, so first law says

$$\text{engine:} \qquad \sum F_y = T_1 + (-w) = 0 \qquad \text{and} \qquad T_1 = w$$

- Horizontal and slanted chains don't exert forces on engine itself as they are not attached to it. In ring fbd, recall that T_1, T_2 and T_3 are magnitudes of forces. Resolving them into their x- and y-components gives us two equations for ring:

$$\text{ring:} \qquad \sum F_x = T_3 \cos 60° + (-T_2) = 0$$

$$\text{ring:} \qquad \sum F_y = T_3 \sin 60° + (-T_1) = 0$$

- Since $T_1 = w$ from engine eq, we can rewrite second ring eq:

$$T_3 = \frac{T_1}{\sin 60°} = \frac{w}{\sin 60°} = 1.2w$$

- Using this result in first ring equation:

$$T_2 = T_3 \cos 60° = w \frac{\cos 60°}{\sin 60°} = 0.58w$$

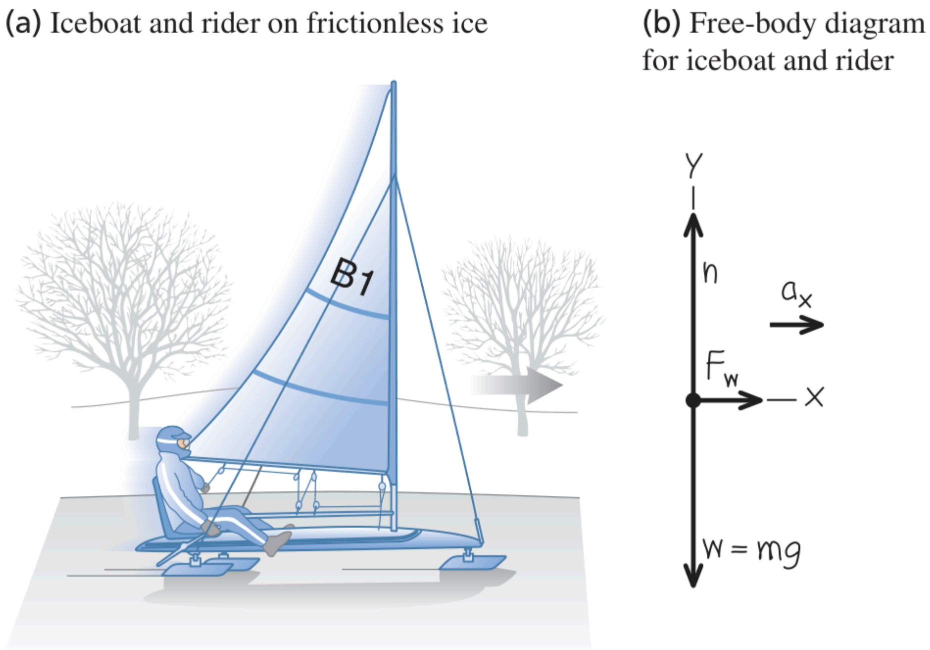
- Key concept: In two-dim problems, always write two force equations for each object: one for the x-components of the forces and one for the y-components of the forces

Using the second law: dynamics of objects 🅱️

$$\sum \vec{F} = m\vec{a}, \qquad \sum F_x = ma_x, \quad \sum F_y = ma_y$$

- If the vector sum of forces on an object is not zero, the object accelerates
  - Its acceleration is related to net force by second law
- Just as for equilibrium problems, free-body diagrams are useful, and the normal force exerted on an object is not always equal to its weight

Example. An iceboat is at rest on a frictionless horizontal surface as in figure. Due to the blowing wind, 4.0 s after the iceboat is released, it is moving to the right at 6.0 m/s (about 22 km/h). What constant horizontal force F_W does the wind exert on the iceboat? The combined mass of iceboat and rider is 200 kg.



- Force F_W is in $+x$-direction while n and $w = mg$ are in $+y$- and $-y$-directions, respectively. Hence we have

$$\sum F_x = F_W = ma_x,$$

$$\sum F_y = n + (-mg) = 0 \qquad \text{so} \qquad n = wg$$

- To find F_W, we first solve for a_x using the kinematic equation

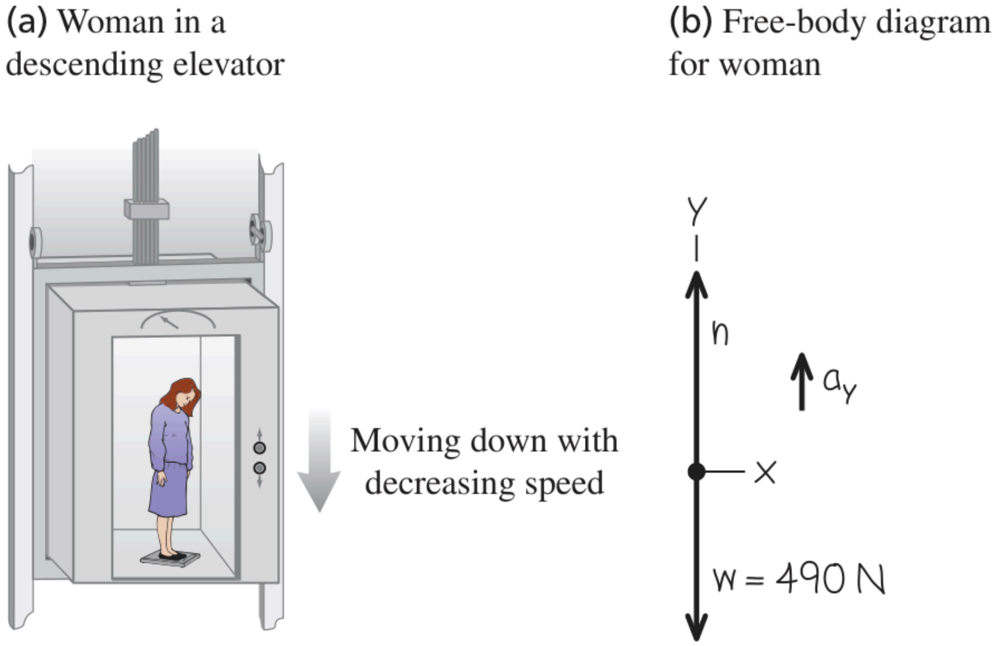
$$v_x = v_{0x} + a_x t$$

$$\implies a_x = \frac{v_x - v_{0x}}{t} = \frac{6.0 \text{ m/s} - 0}{4.0 \text{ s}} = 1.5 \text{ m/s}^2$$

$$\implies F_W = ma_x = (200 \text{ kg})(1.5 \text{ m/s}^2) = 300 \text{ kg m/s}^2 = 300 \text{ N}$$

- Key concept: It's usually best to choose one positive axis to be in the direction of the acceleration

Example. A 50.0 kg woman stands on a bathroom scale while riding in an elevator. The elevator is initially moving downward at 10.0 m/s. It slows to a stop with constant acceleration in a distance of 25.0 m. While the elevator is moving downward with decreasing speed, what is the reading on the scale?



- We start by finding a_y using the kinematic equation

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$\implies a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(0)^2 - (-10.0 \text{ m/s})^2}{2(-25.0 \text{ m})} = +2.00 \text{ m/s}^2$$

- The acceleration is upward (positive), just as it should be

- Second law then gives

$$\sum F_y = n + (-mg) = ma_y \qquad \implies n = mg + ma_y = m(g + a_y)$$

$$= (50.0 \text{ kg})(9.80 \text{ m/s}^2 + 2.00 \text{ m/s}^2) \qquad = 590 \text{ N}$$

- Key concept: When riding an accelerating vehicle such as an elevator, your **apparent weight** (the normal force that vehicle exerts on you) is in general not equal to your actual weight

Questions? 🙄

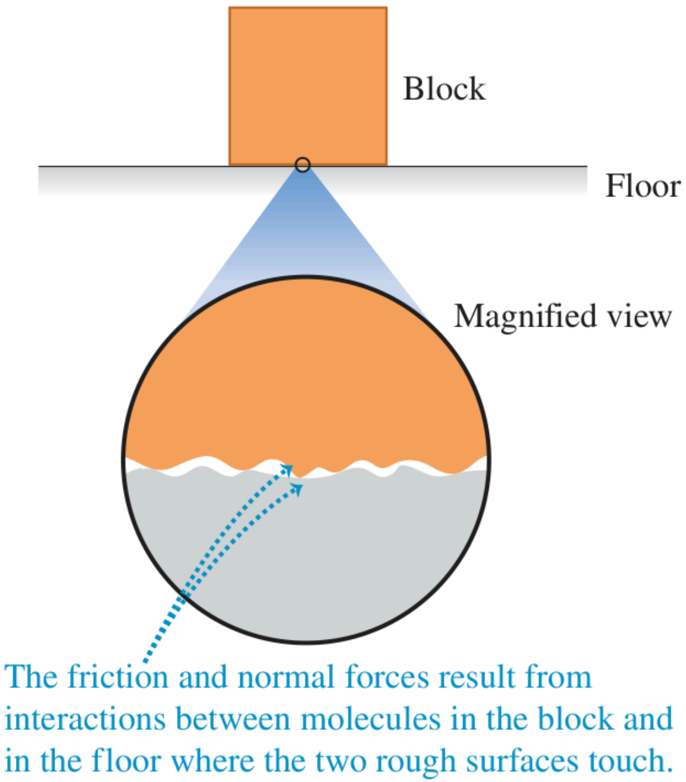
Friction forces 🐌

Friction

- Friction is important in many aspects of everyday life
  - eg. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car 🚗
- Air drag, the friction force exerted by the air on an object moving through it, decreases automotive fuel economy but makes parachutes work 🪂

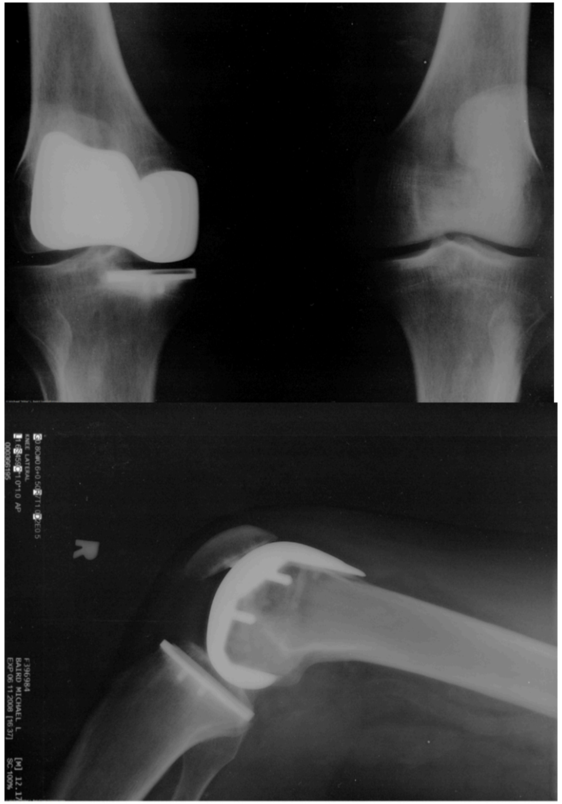
Friction

- On a microscopic level, friction and normal forces result from intermolecular forces (electrical in nature) between two rough surfaces at points where they come into contact



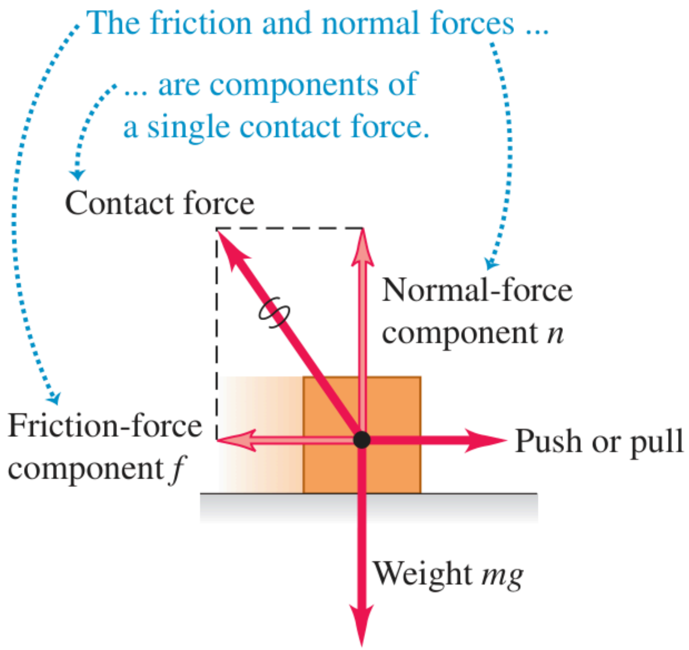
Friction in body

- The joints produce synovial fluid that reduces friction and wear between ends of two bones
- A damaged or arthritic joint can be replaced by artificial joints made of metals or plastic, also with very small coefficients of friction



Friction as component

- The contact force between two objects can always be represented in terms of a normal force $\vec{n}$ perpendicular to surface of contact and a **friction force** $\vec{f}$ parallel to surface



Kinetic friction

- When an object is sliding over the surface, the friction force is called **kinetic friction** $\vec{\boldsymbol{f}}_{\boldsymbol{k}}$. Its magnitude f_k is approximately

$$f_k = \mu_k n$$

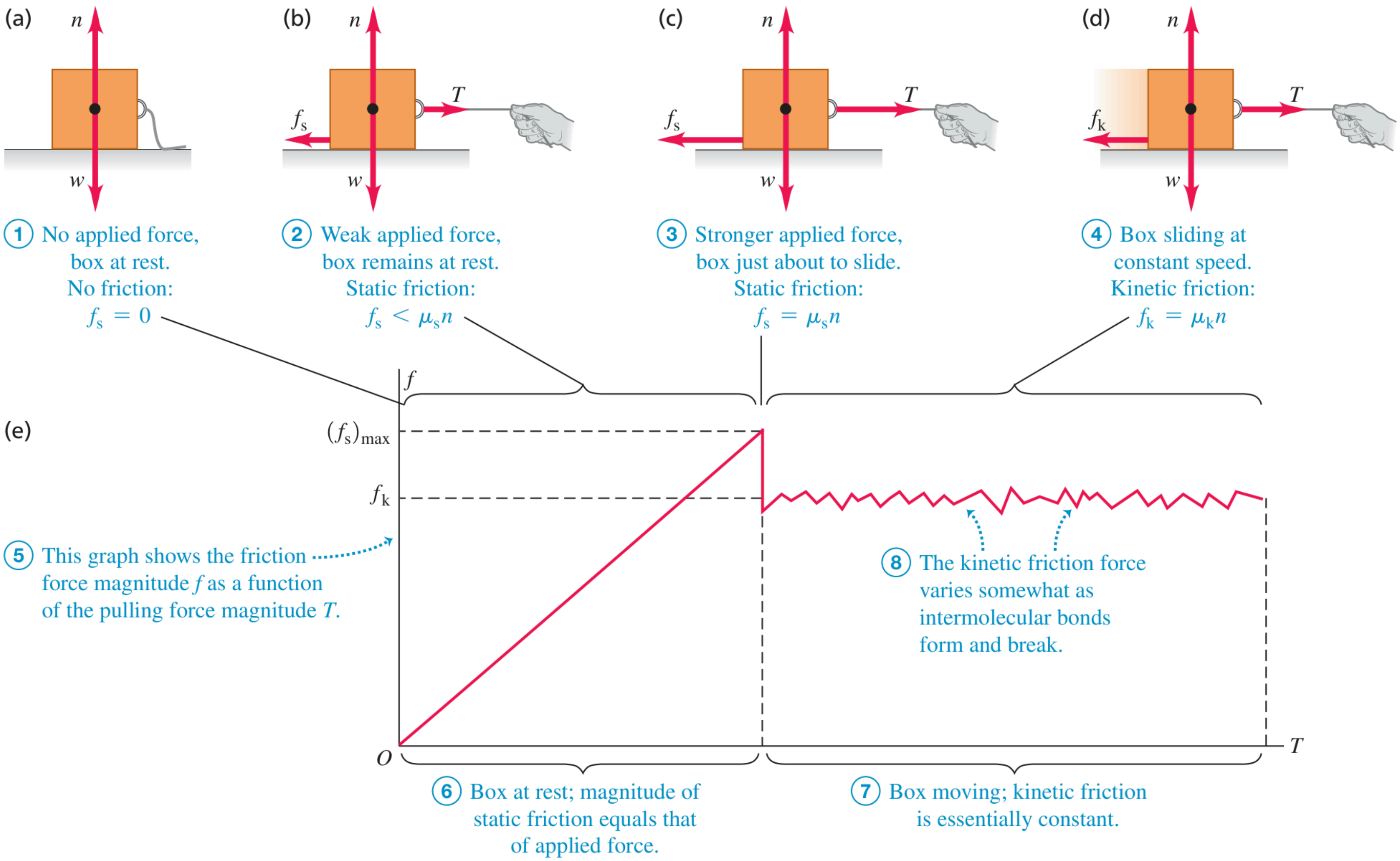
  where μ_k is the coefficient of kinetic friction
  - The more slippery the surface, the smaller this coefficient
- Friction can also depend on speed of object relative to surface
  - For now, we'll ignore this effect and assume that μ_k and f_k are independent of speed

Static friction

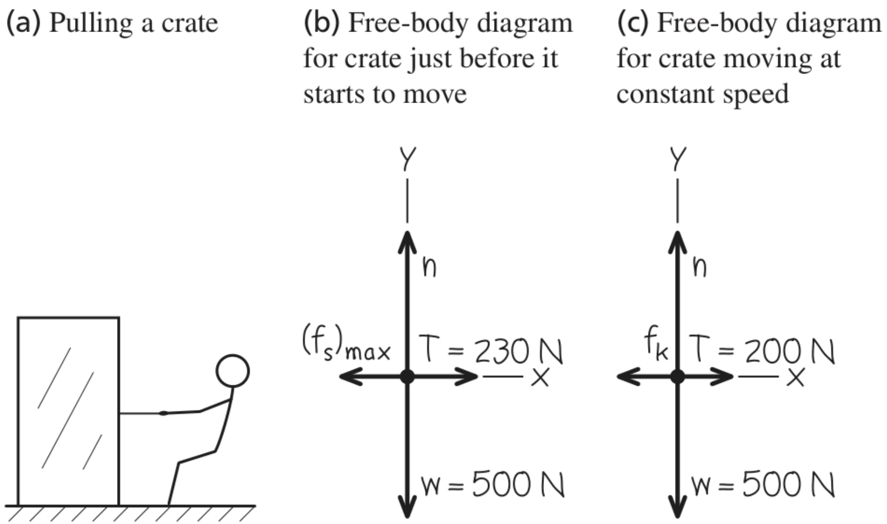
- When an object is not moving relative to a surface, the friction force is called **static friction** $\vec{\boldsymbol{f}}_{\boldsymbol{s}}$
  - Its magnitude, which may be anything from zero to some maximum value depending on situation, is approximately

$$f_s \leq f_{s,\,\mathrm{max}} = \mu_s n$$

    where μ_s is the coefficient of static friction
  - Usually $\mu_s > \mu_k$ for a given pair of surfaces in contact


(a)
n
w
1 No applied force, box at rest. No friction: $f_s = 0$
(b)
n
T
f_s
w
2 Weak applied force, box remains at rest. Static friction: $f_s < \mu_s n$
(c)
n
T
f_s
w
3 Stronger applied force, box just about to slide. Static friction: $f_s = \mu_s n$
(d)
n
T
f_k
w
4 Box sliding at constant speed. Kinetic friction: $f_k = \mu_k n$
(e)
f
$(f_s)_{max}$
f_k
O
T
5 This graph shows the friction force magnitude f as a function of the pulling force magnitude T.
8 The kinetic friction force varies somewhat as intermolecular bonds form and break.
6 Box at rest; magnitude of static friction equals that of applied force.
7 Box moving; kinetic friction is essentially constant.

Example. You want to move a 500 N crate across a level floor. To start the crate moving, you have to pull with a 230 N horizontal force. Once the crate starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?



- Just before crate starts to move, we have via first law

$$\sum F_x = T + (-f_{s,\,\max}) = 0 \qquad \text{so} \qquad f_{s,\,\max} = T = 230 \text{ N}$$

$$\sum F_y = n + (-w) = 0 \qquad \text{so} \qquad n = w = 500 \text{ N}$$

- Solving for μ_s, we get

$$\mu_s = \frac{f_{s,\,\max}}{n} = \frac{230\ \text{N}}{500\ \text{N}} = 0.46$$

- After crate starts to move (constant $\vec{v}$), we have via first law

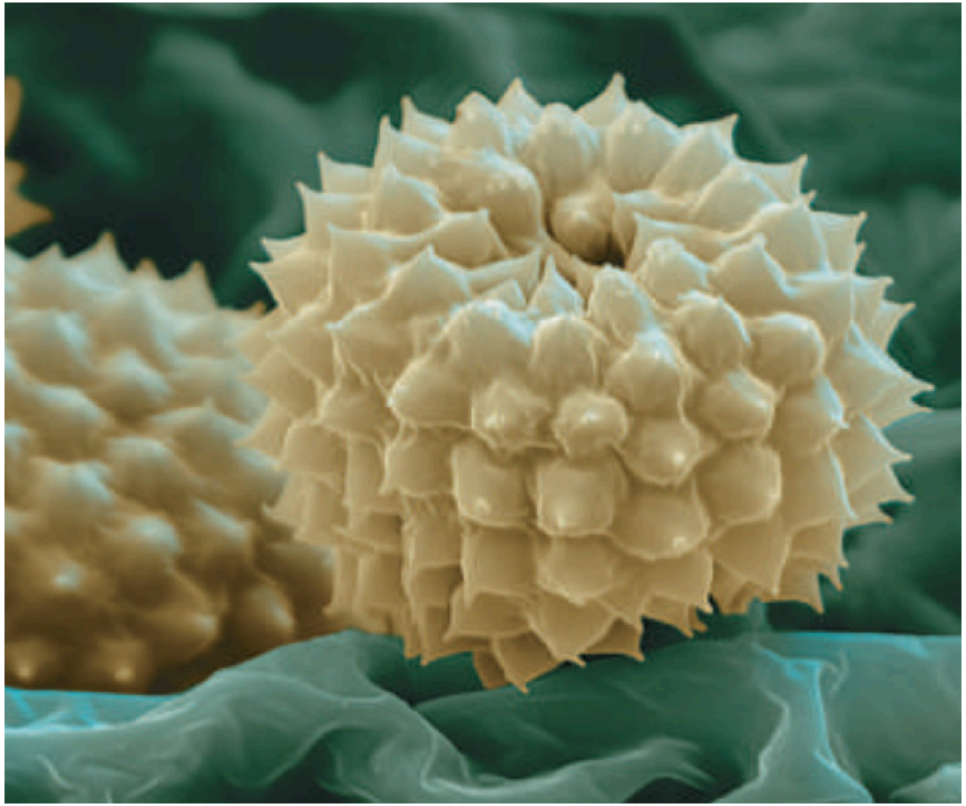
$$\sum F_x = T + (-f_k) = 0 \qquad \text{so} \qquad f_k = T = 200\ \text{N}$$
$$\sum F_y = n + (-w) = 0 \qquad \text{so} \qquad n = w = 500\ \text{N}$$

- Computing for μ_k, we find

$$\mu_k = \frac{f_k}{n} = \frac{200\ \text{N}}{500\ \text{N}} = 0.40$$

Fluid resistance

- The force that a fluid (a gas or liquid) exerts on an object moving through it is called **force of fluid resistance** $\boldsymbol{f}$
  - eg. ⟵🚗 🪟 🤳
- The moving object exerts force on fluid to push it out of way. By third law, fluid pushes back on the object



Fluid resistance

- For small objects moving at low speed v, its magnitude is

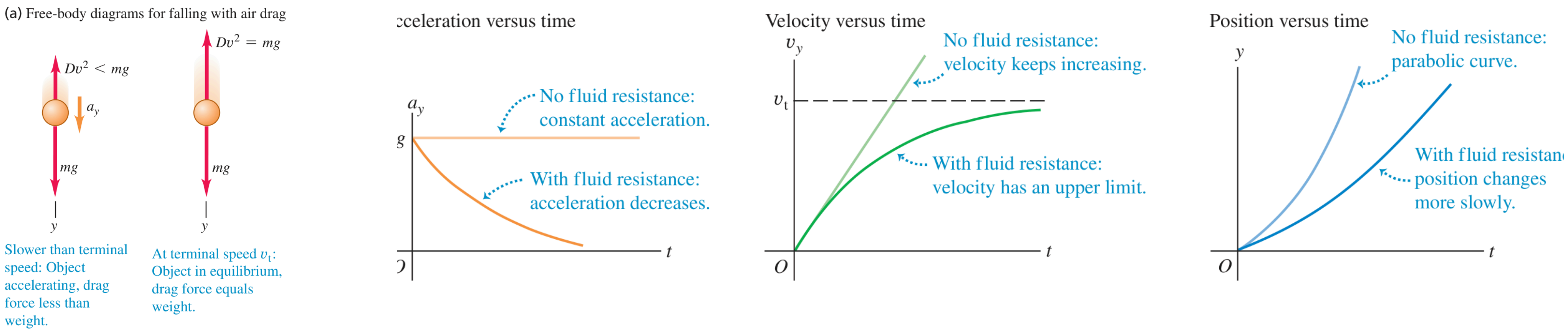
$$f = kv$$

  - Alternatively, using Stokes' law, $f = (6\pi\eta r)v$, where r is the radius of object and η is fluid viscosity. Above k has unit kg/s
- For larger objects moving thru air at speed of a tossed tennis ball or faster, resisting force is called **air drag**, or simply **drag**:

$$f = Dv^2$$

- Alternatively, $f = \left(\frac{1}{2}\rho AC\right)v^2$, where C is drag coefficient, A is area of the object facing the fluid, and ρ is the fluid density. Above D has unit kg/m

- As speed increases, resisting force also increases, until finally it is equal in magnitude to weight, wherein $a = 0$ and there is no further increase in speed. The final speed v_t is **terminal speed**



Example. For a human body falling through air in a spread-eagle position, the numerical value of the constant D is about 0.25 kg/m. Find the terminal speed for a 50 kg skydiver.

$$v_t = \sqrt{\frac{mg}{D}}$$

$$= \sqrt{\frac{(50\text{ kg})(9.80\text{ m/s}^2)}{0.25\text{ kg/m}}}$$

$$= 44\text{ m/s (about 160 km/h)}$$

Questions? 🤓

Elasticity 🪱

Deformation

- The rigid body is a useful idealized model, but **deformations** like stretching, squeezing, and twisting of real objects when forces are applied are often too important to ignore
- **Hooke's law** states that in elastic deformations, **stress** (force per unit area) is proportional to **strain** (fractional deformation)
  - The proportionality constant is called the **elastic modulus**

$$\frac{\text{stress}}{\text{strain}} = \text{elastic modulus}$$

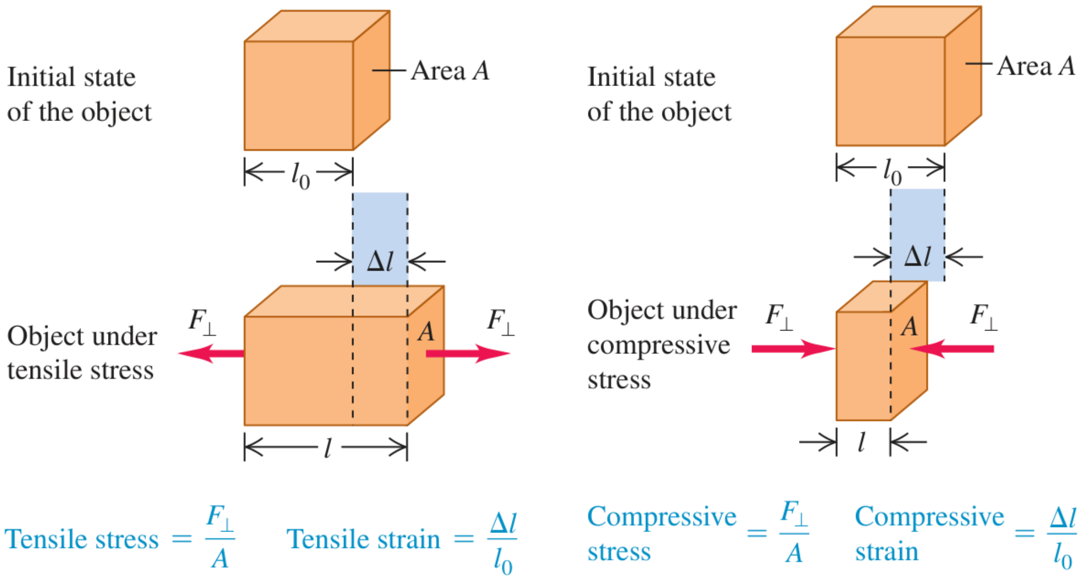
- Examples of stress
  - 🎸 strings under tensile stress, being stretched by forces acting at their ends
  - 🤿 under bulk stress, being squeezed from all sides by forces due to water pressure
  - 🎀 under shear stress, being deformed and eventually cut by forces exerted by the scissors



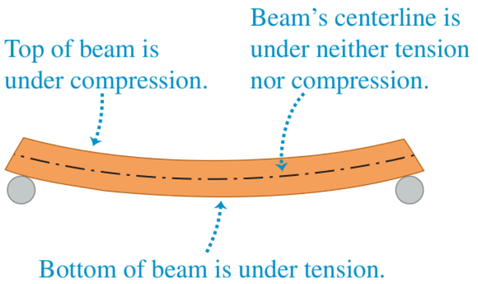
Tensile and compressive stress

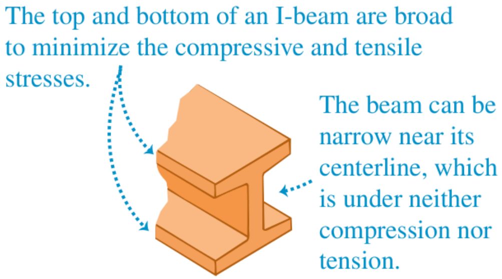
- **Tensile stress** is tensile force per unit area $F_\perp/A$
- **Tensile strain** is fractional change in length $\Delta l/l_0$
- The elastic modulus for tension is called **Young's modulus $\boldsymbol{Y}$**
- Compressive stress and strain are defined in the same way

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F_\perp/A}{\Delta l/l_0} = \frac{F_\perp}{A}\frac{l_0}{\Delta l}$$

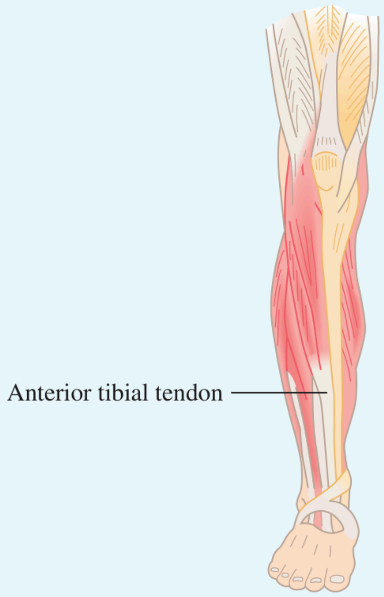


(a) (b)





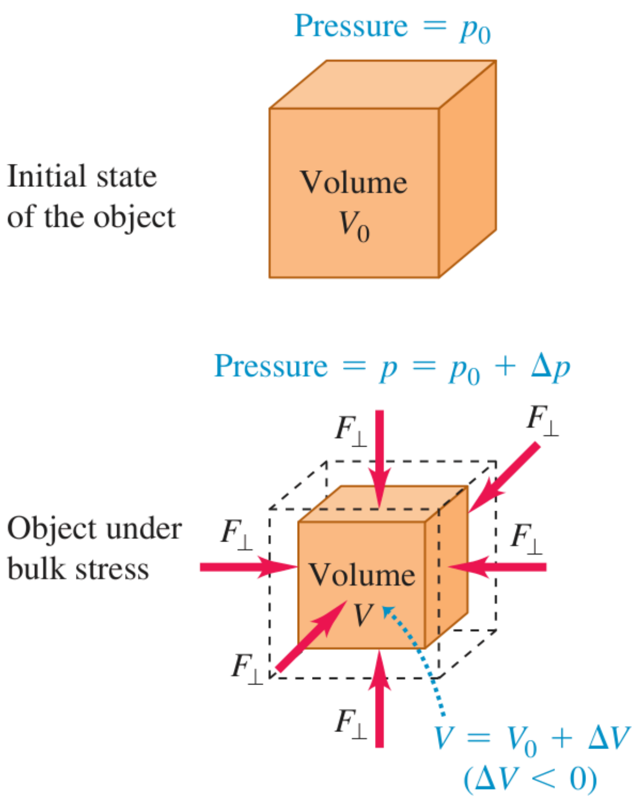
BIO APPLICATION Young's Modulus of a Tendon The anterior tibial tendon connects your foot to the large muscle that runs along the side of your shinbone. (You can feel this tendon at the front of your ankle.) Measurements show that this tendon has a Young's modulus of 1.2×10^9 Pa, much less than for the metals listed in Table 11.1. Hence this tendon stretches substantially (up to 2.5% of its length) in response to the stresses experienced in walking and running.



Bulk stress

- Pressure in a fluid is force per unit area
- **Bulk stress** is pressure change Δp, and **bulk strain** is fractional volume change $\Delta V/V_0$
- The elastic modulus for compression is called **bulk modulus $\boldsymbol{B}$**
- **Compressibility $\boldsymbol{k}$** is the reciprocal of bulk modulus: $k = 1/B$

$$p = \frac{F_\perp}{A}, \qquad B = \frac{\text{bulk stress}}{\text{bulk strain}} = \frac{\Delta p}{\Delta V/V_0}$$



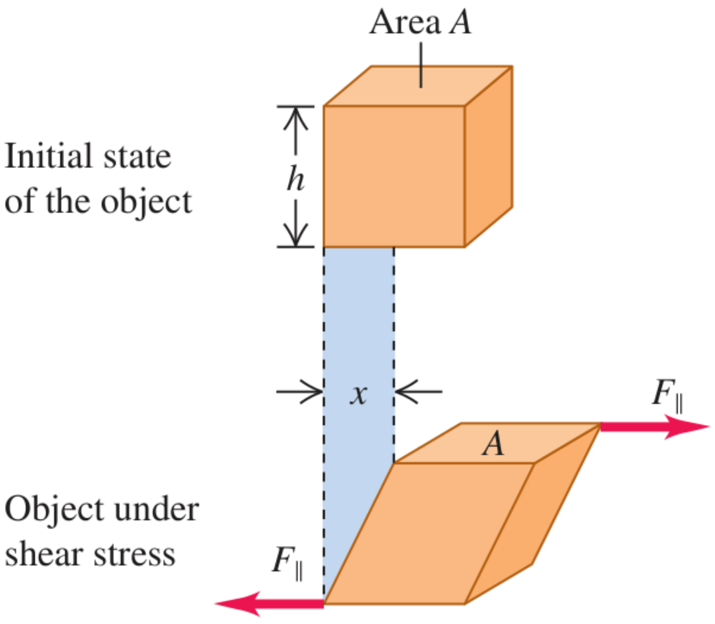
Bulk stress $= \Delta p$ Bulk strain $= \dfrac{\Delta V}{V_0}$

BIO APPLICATION Bulk Stress on an Anglerfish The anglerfish (*Melanocetus johnsonii*) is found in oceans throughout the world at depths as great as 1000 m, where the pressure (that is, the bulk stress) is about 100 atmospheres. Anglerfish are able to withstand such stress because they have no internal air spaces, unlike fish found in the upper ocean, where pressures are lower. The largest anglerfish are about 12 cm (5 in.) long.

Shear stress

- **Shear stress** is force per unit area $F_{\parallel}/A$ for a force applied tangent to a surface
- **Shear strain** is the displacement x of one side divided by the transverse dimension h
- The elastic modulus for shear is called the **shear modulus S**

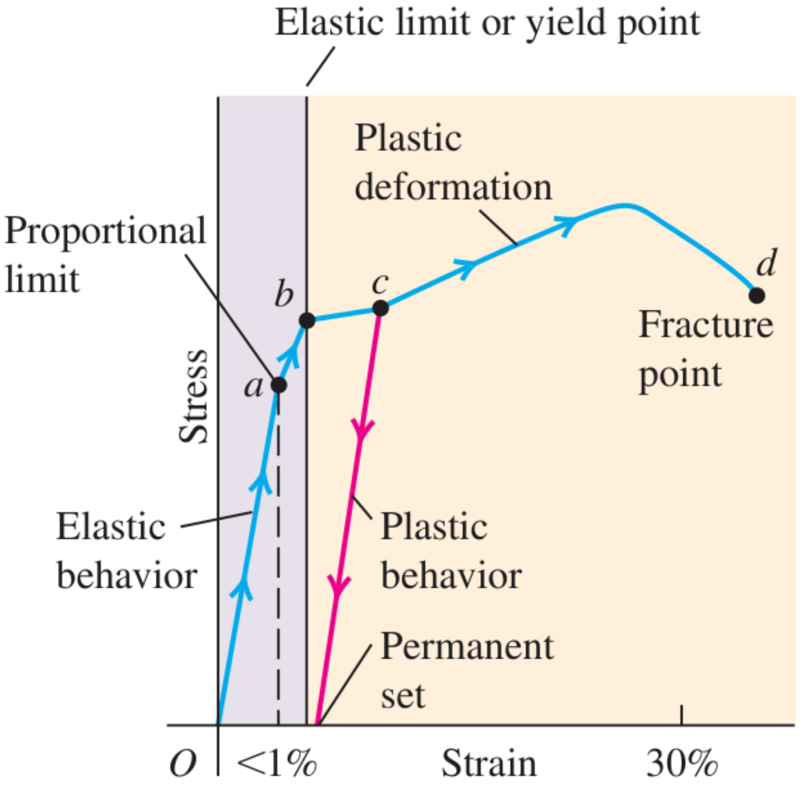
$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F_{\parallel}/A}{x/h} = \frac{F_{\parallel}}{A}\frac{h}{x}$$

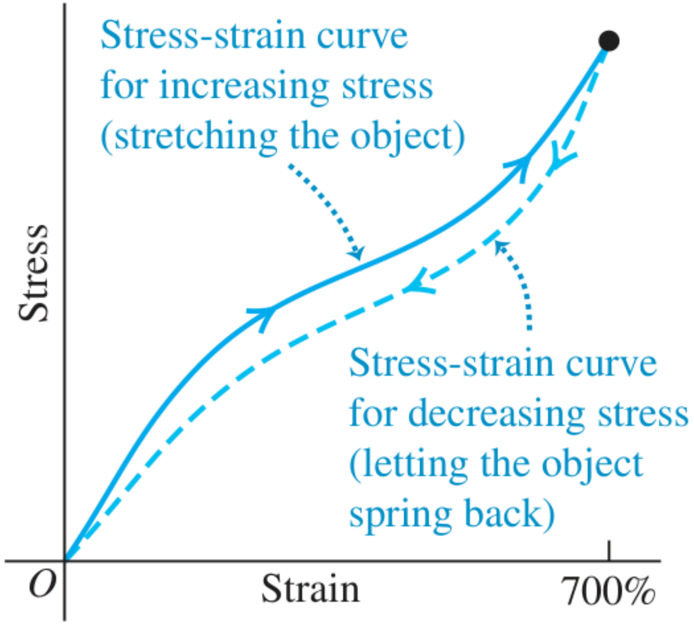


$$\text{Shear stress} = \frac{F_{\parallel}}{A} \qquad \text{Shear strain} = \frac{x}{h}$$

Limits of Hooke's law

- The **proportional limit** is the maximum stress for which stress and strain are proportional
  - Beyond the proportional limit, Hooke's law is not valid
- The **elastic limit** is the stress beyond which irreversible deformation occurs
- The **breaking stress**, or ultimate strength, is the stress at which the material breaks


Elastic limit or yield point
Plastic
deformation
Proportional
limit
b
c
d
Fracture
point
a
Stress
Elastic
behavior
Plastic
behavior
Permanent
set
O
<1%
Strain
30%


Stress-strain curve
for increasing stress
(stretching the object)
Stress
Stress-strain curve
for decreasing stress
(letting the object
spring back)
O
Strain
700%

Questions? 🙄

Quiz time 🕐

Hell nah, that steel post wasn't there 🅿️

While parking your car, you accidentally back into a steel post. You pull forward until the car no longer touches the post and then get out to inspect the damage. What does your rear bumper look like if the strain in the impact was

- (a) less than at the proportional limit
- (b) greater than at proportional lim but less than at yield point
- (c) greater than at yield point but less than at fracture point
- (d) greater than at the fracture point?