Kinematics, reviewed

R. Torres

2025 W36[1]

[1]Phys 20.01 Mod 1. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Comprehension 📕

Conceptual ⚙️

Problem-solving 🧮

Comprehension 📕

Comprehension #7

Example. Which of the following expressions correctly illustrates the SI units of each one of the variables in the formula?

$$m\Delta v = F\Delta t$$

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$$m\Delta v = F\Delta t$$

a. $\text{lb} \times \text{mph} = \text{ft} \times \text{lb} \times \text{s}$
b. $\text{lb} \times \text{km} = \text{N} \times \text{s}$
c. $\text{kg} \times \text{m/s} = \text{N} \times \text{s}$
d. $\text{g} \times \text{m/s} = \text{N} \times \text{s}$

Comprehension #7

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- **Answer: $\mathbf{kg \times m/s = N \times s}$**

Comprehension #19

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Comprehension #19

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a. You are driving a car toward the east and your friend drives past you in opposite direction with same speed. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
b. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
c. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the west. In your friend's frame of reference, he will be at rest.
d. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the east. In your friend's frame of reference, he will be at rest.

Comprehension #19

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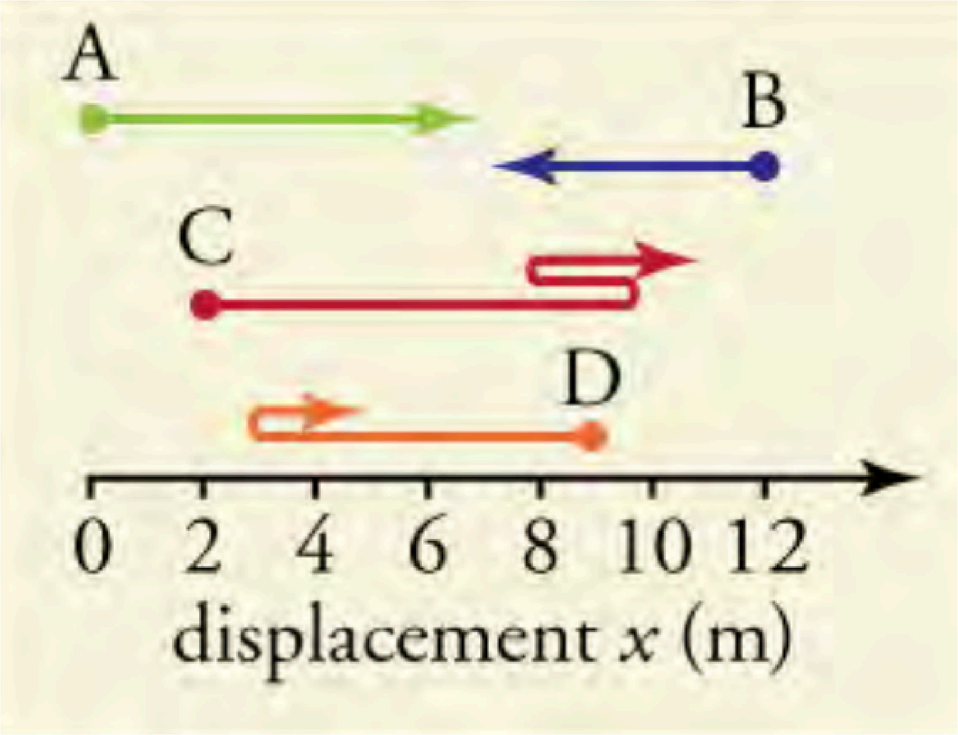
- Hint 2: Remember that "at rest" means not moving relative to something. What is each person not moving relative to?
- **Answer: You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the west. In your friend's frame of reference, he will be at rest.**

Conceptual ⚙️

Conceptual #1

Example. Find the distance traveled from the starting point for each path. Which path has the maximum distance?

a. A
b. B
c. C
d. D



Conceptual #1

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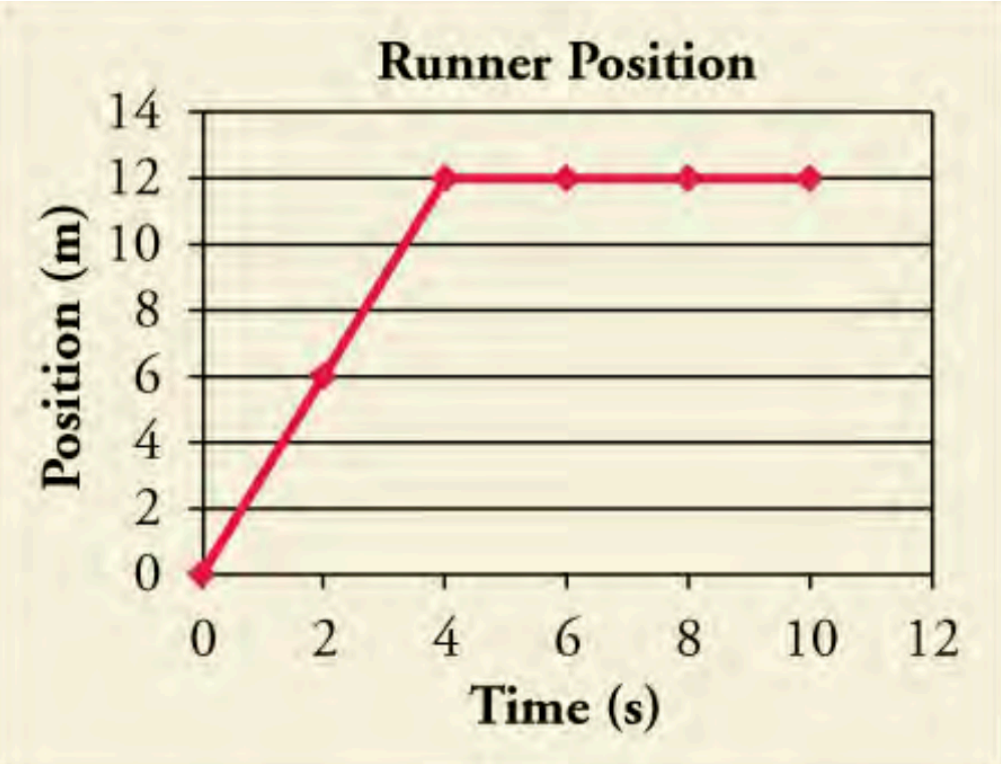
- **Answer: C**

Conceptual #7

Example. Use the graph to describe what the runner's motion looks like. How are average velocity for only the first four seconds and instantaneous velocity related? What is the runner's net displacement over the time shown?



Conceptual #7

a. The net displacement is 12 m and average velocity is equal to the instantaneous velocity
b. The net displacement is 12 m and average velocity is two times the instantaneous velocity
c. The net displacement is $10 + 12 = 22$ m and average velocity is equal to the instantaneous velocity
d. The net displacement is $10 + 12 = 22$ m and average velocity is two times the instantaneous velocity

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- **Answer: The net displacement is 12 m and average velocity is equal to the instantaneous velocity**

Conceptual #8

Example. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?

a. 0.574 s
b. 0.956 s
c. 1.53 s
d. 1.91 s

Conceptual #8

- Hint 1: A sketch

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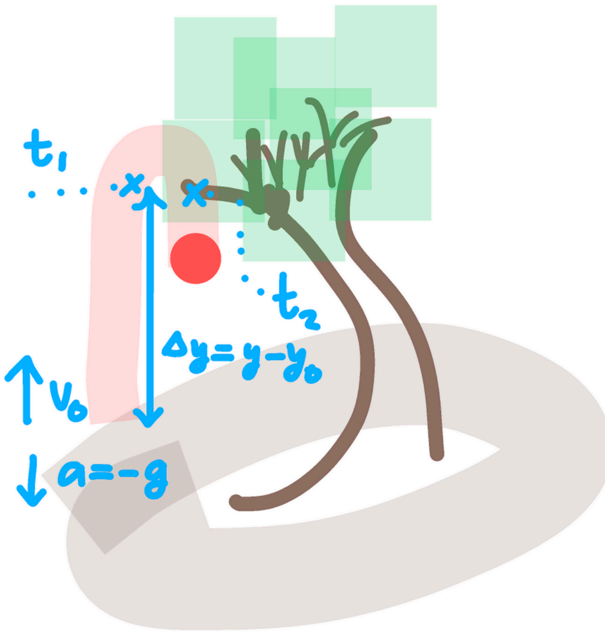

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Conceptual #8

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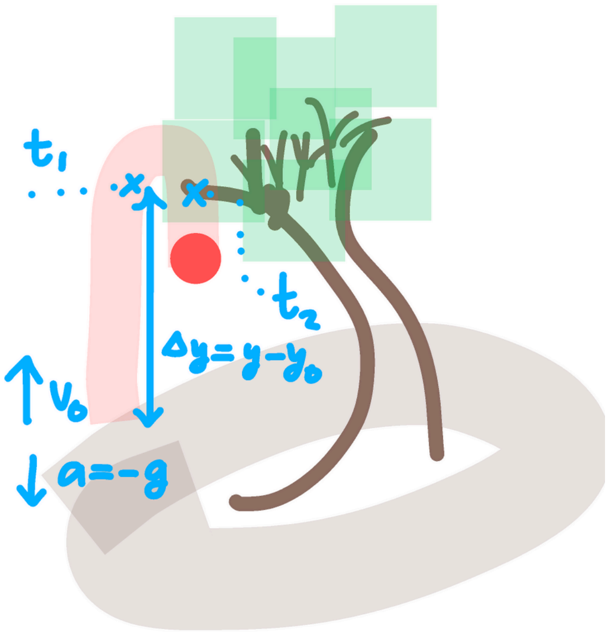

- Hint 2: Use kinematic equation $y = y_0 + v_0 t + \frac{1}{2}at^2$. The ball passes by $\Delta y = 7$ m tall branch twice (going up, going down), so there are two times t_1 and t_2. From above equation, isolate then solve t
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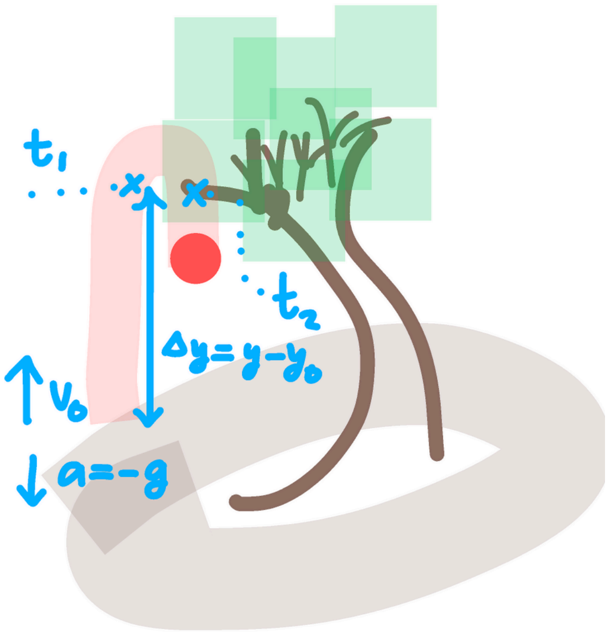

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- **Answer: 1.91 s**

Conceptual #14

Example. At a place where g is 9.8 m/s^2, an object is thrown vertically downward with a speed of 10 m/s while a different object is thrown vertically upward with a speed of 20 m/s. Which object undergoes a greater change in speed in a time of 2 s?

a. The first object because the speed vector points in the same direction as the acceleration due to gravity
b. The second object because it has a higher velocity
c. Both objects undergo the same change in speed
d. Cannot be determined from the information given

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- **Answer: Both objects undergo same change in speed**

Conceptual #16

Example. A base jumper runs off a cliff, with a speed of 3 m/s. What is its overall velocity after 5 s?

a. 3 m/s
b. −5 m/s
c. 5 m/s
d. 10 m/s

Conceptual #16

- Correction: ...after 0.5 s, not ...after 5 s
- Note: uses two dimensions, and is not convered in exam
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- **Answer: 5 m/s (ish), it's actually 5.75 m/s**

Conceptual #20

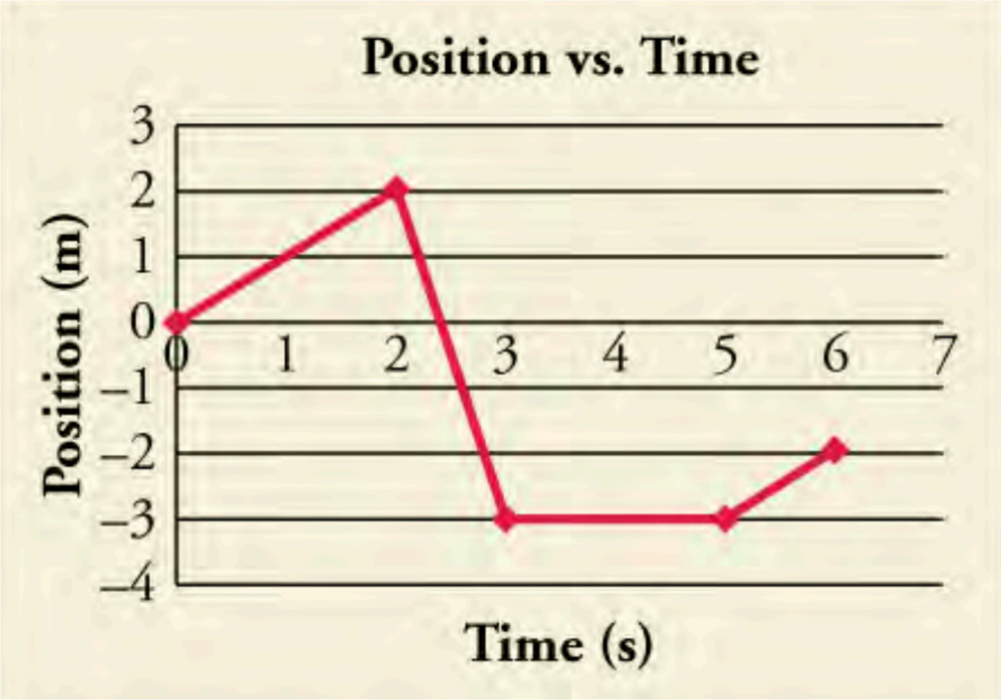
Example. What is the average velocity for the whole time period shown in the graph?

a. $-1/3$ m/s

b. $-3/4$ m/s

c. $1/3$ m/s

d. $3/4$ m/s



Conceptual #20

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-

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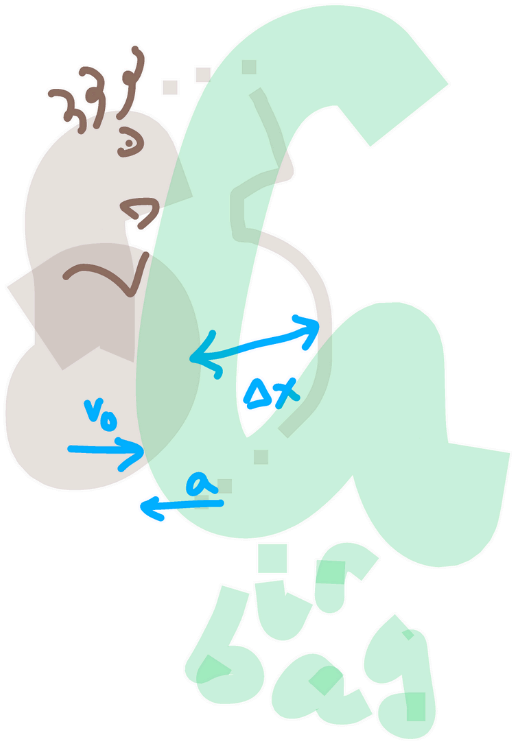
- Hint 2: $\Delta x = x_f - x_0$ where x_0 is initial and x_f is final
- **Answer: $-1/3$ m/s**

Problem-solving 🧮

Problem solving #1

Example. **Airbag injuries.** During an auto accident, the vehicle's airbags deploy and slow down the passengers more gently than if they had hit the windshield or steering wheel. According to safety standards, airbags produce a maximum acceleration of $60g$ that lasts for only 36 ms (or less). How far (in meters) does a person travel in coming to a complete stop in 36 ms at a constant acceleration of $60g$?

Problem solving #1



- Hint 1: What does $60g$ mean in terms of m/s^2? ($g = 9.8\frac{\text{m}}{\text{s}^2}$)
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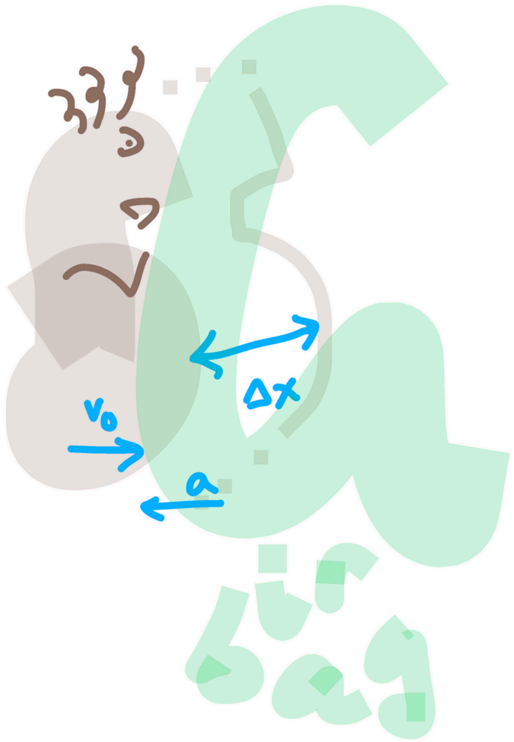

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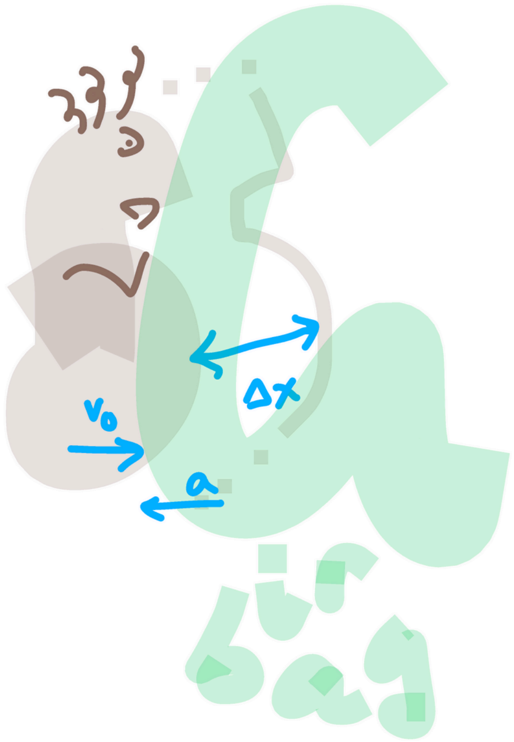

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- Hint 2: What is the final velocity v?
- Hint 3: Which kinematic equation relates velocity, acceleration, displacement?
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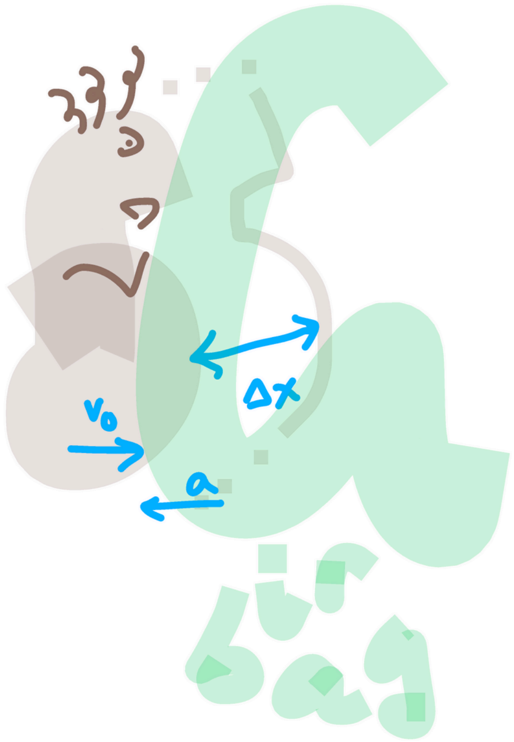

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- Hint 2: What is the final velocity v?
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- Hint 4: Make sure units are consistent (meters, seconds)

Problem solving #1

- Calculate and convert
  - $a = 60g = 60(9.8 \text{ m/s}^2) = 588 \text{ m/s}^2$
  - $t = 36 \text{ ms} = 36 \times 10^{-3} \text{ s} = 0.036 \text{ s}$ (here, ms is milliseconds)
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- Listing down knowns and unknowns
  - $a = -588 \mathrm{m/s^2}$ (deceleration, as it slows down object)
  - $t = 0.036\ \mathrm{s}$
  - $v = v_f = 0$ (object comes to a complete stop)
  - $\Delta x = ?$

Problem solving #1

- Probably relevant equations. Recall $\Delta x = x_f - x_0 = x - x_0$
  - $v = v_0 + at$
  - $v^2 = v_0^2 + 2a\Delta x$
  - $\Delta x = v_0 t + \frac{1}{2}at^2$
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  - $v = v_0 + at$
  - $v^2 = v_0^2 + 2a\Delta x$
  - $\Delta x = v_0 t + \frac{1}{2}at^2$
- Finding initial velocity v_0, we get

$$\begin{aligned} v = v_0 + at \implies v_0 &= v - at \qquad \text{(subtract } at\text{)} \\ &= 0 - \left(-588\ \text{m/s}^2\right)(0.036\ \text{s}) \\ &= 21.168\ \text{m/s} \end{aligned}$$

Problem solving #1

- Solving how far (displacement Δx), using method 1, we get

$$
\begin{aligned}
\Delta x &= v_0 t + \frac{1}{2} a t^2 \\
&= (21.168 \text{ m/s})(0.036 \text{ s}) + \frac{1}{2}(-588 \text{ m/s}^2)(0.036 \text{ s})^2 \\
&= 0.762 \frac{\text{m}}{\cancel{\text{s}}} \cancel{\text{s}} - 0.381 \frac{\text{m}}{\cancel{\text{s}^2}} \cancel{\text{s}^2} \\
&= 0.381 \text{ m}
\end{aligned}
$$

Problem solving #1

- Solving how far using method 2, we get

$$
\begin{aligned}
v^2 &= v_0^2 + 2a\Delta x \\
\implies v^2 - v_0^2 &= 2a\Delta x && \text{(subtract } v_0^2\text{)} \\
\implies \Delta x &= \frac{v^2 - v_0^2}{2a} && \text{(divide by } 2a\text{)} \\
&= \frac{\left(0^2 - (21.168)^2\right)\frac{\mathrm{m}^2}{\mathrm{s}^2}}{2\left(-588\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)} = \frac{448.084\,\frac{\mathrm{m}^{\cancel{2}}}{\cancel{\mathrm{s}^2}}}{-1176\,\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}} = 0.381\ \mathrm{m}
\end{aligned}
$$

Problem solving #1

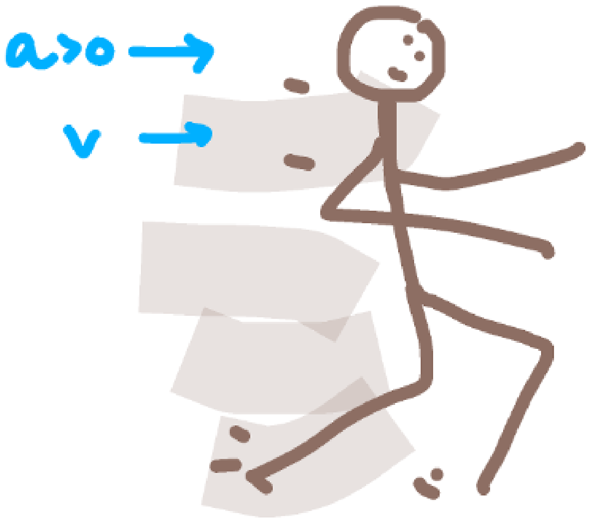
The person travels (or is displaced) approximately 0.381 m before coming to a complete stop

Problem solving #2

Example. A typical male sprinter can maintain his maximum acceleration for 2.0 s, and his maximum speed is 10 m/s. After he reaches this maximum speed, his acceleration becomes zero, and then he runs at constant speed. Assume that his acceleration is constant during the first 2.0 s of the race, that he starts from rest, and that he runs in a straight line. (a) How far has the sprinter run when he reaches his maximum speed? (b) What is the magnitude of his average velocity for a race of these lengths: 50.0 m, 100.0 m, 200.0 m?

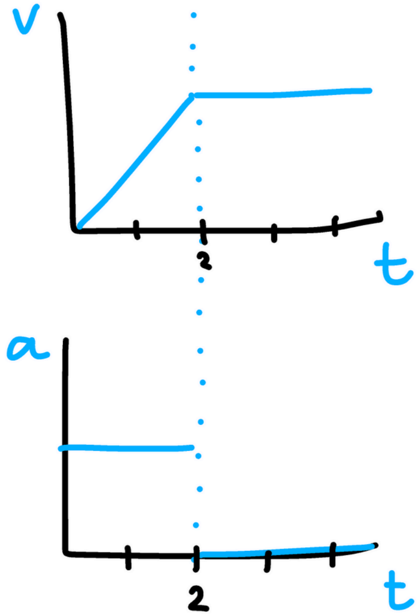
Problem solving #2

- Hint 1: The sprint has two parts



accelerating

not accelerating



Problem solving #2

- Hint 1: You already know the time and distance during the acceleration phase. How can you calculate the time spent at constant velocity (second part of sprint) for each race length?
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Problem solving #2

- Hint 1: You already know the time and distance during the acceleration phase. How can you calculate the time spent at constant velocity (second part of sprint) for each race length?
- Hint 2: total time = time accelerating + time at constant v
- Hint 3: For each race length, calculate the distance traveled at constant velocity. Then use the constant velocity to find the time spent at that velocity

Problem solving #2

- Listing down knowns and unknowns
  - $t = 2.0$
  - $v = 10$ m/s
  - $v_0 = 0$ (starts from rest)
  - $\Delta x =?$
-

Problem solving #2

- Listing down knowns and unknowns
  - $t = 2.0$
  - $v = 10$ m/s
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  - $\Delta x = ?$
- Surely relevant equations. Recall $\Delta x = x - x_0$
  - $v = v_0 + at$
  - $\Delta x = v_0 t + \frac{1}{2}at^2$
  - $\overline{v} = \Delta x / \Delta t$

Problem solving #2

- Calculating acceleration a, we get

$$v = v_0 + at \implies v - v_0 = at \qquad \text{(subtract } v_0\text{)}$$

$$\implies a = \frac{v - v_0}{t} \qquad \text{(divide by } t\text{)}$$

$$= \frac{(10 - 0)\,\frac{\text{m}}{\text{s}}}{2.0\ \text{s}}$$

$$= 5.0\ \text{m/s}^2$$

Problem solving #2

- Finding distance (distance = displacement because the sprinter doesn't change direction and moves only in a straight line)

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$= (0 \text{ m/s})(2.0 \text{ s}) + \frac{1}{2}(5.0 \text{ m/s}^2)(2.0 \text{ s})^2$$

$$= \frac{1}{2}\left(5.0 \frac{\text{m}}{\cancel{\text{s}^2}}\right)(4.0 \ \cancel{\text{s}^2}) \qquad = 10.0 \ \text{ m}$$

  - The sprinter has run 10.0 m when he reaches his max speed

Problem solving #2

- Solving average velocity for 50.0 m race
  - distance traveled at constant v: 50.0 m − 10.0 m = 40.0 m
  - time spent at constant v:
    $t = \Delta x / v = (40.0\text{ m})/(10.0\text{ m/s}) = 4.0\text{ s}$
  - total time for race: 2.0 s + 4.0 s = 6.0 s
  - average velocity: 50.0 m/6.0 s = 8.33 m/s

Problem solving #2

- For 100.0 m race
  - distance traveled at constant v: $100.0\text{ m} - 10.0\text{ m} = 90.0\text{ m}$
  - time spent at constant v: $t = (90.0\text{ m})/(10.0\text{ m/s}) = 9.0\text{ s}$
  - total time for race: $2.0\text{ s} + 9.0\text{ s} = 11.0\text{ s}$
  - average velocity: $100.0\text{ m}/11.0\text{ s} = 9.09\text{ m/s}$

Problem solving #2

- For 200.0 m race
  - distance traveled at constant v: 200.0 m − 10.0 m = 190.0 m
  - time spent at constant v: t = (190.0 m)/(10.0 m/s) = 19.0 s
  - total time for race: 2.0 s + 19.0 s = 21.0 s
  - average velocity: 200.0 m/21.0 s = 9.52 m/s

Questions? 🙄