# Problem solving and graphical analysis

R. Torres

2025 W35[1]

[1]Phys 20.01 Mod 1. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

# Agenda

Previously ⏪

Problem solving 🧮

Graphical analysis 📈

Some notes 📎

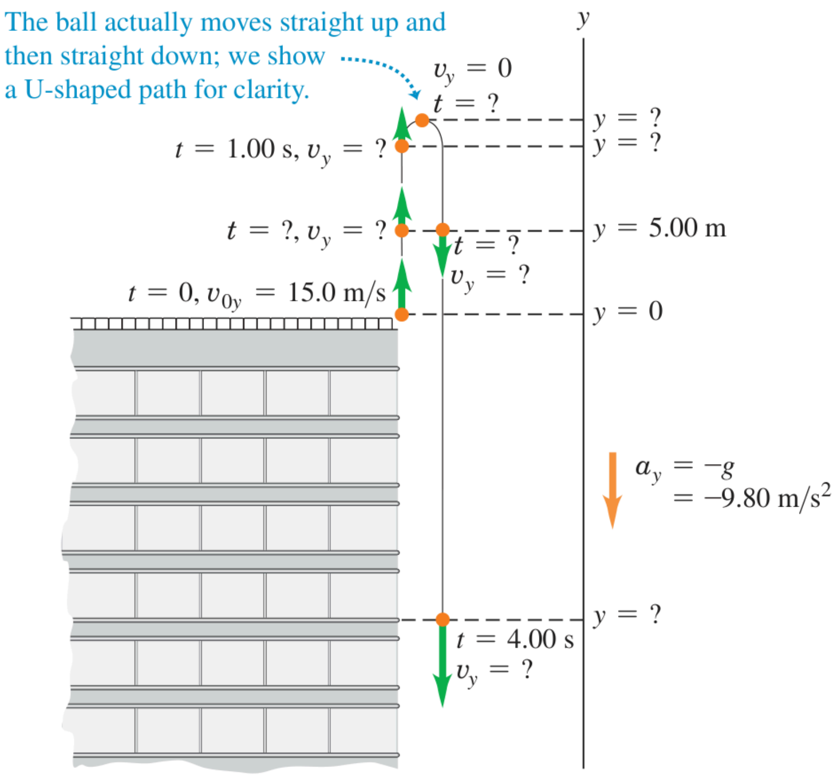
Quiz time 🕐

# Previously ⏪

Motion with constant acceleration, kinematic equations, free-fall

# Up and down motion in free fall

*Example.* You throw a ball vertically upward from roof of a building. The ball leaves your hand with an upward speed of 15.0 m/s and is then in free fall. On its way back down, it just misses the railing. Find ball's position and velocity 1.00 s and 4.00 s after leaving your hand.



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$$y = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + (15.0\ \mathrm{m/s})t + \frac{1}{2}(-9.8\ \mathrm{m/s^2})t^2,$$

$$v = v_0 + at = 15.0\ \mathrm{m/s} + (-9.8\ \mathrm{m/s^2})t.$$

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When $t = 4.00$ s, we get $y = -18.4$ m and $v = -24.2$ m/s.

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When $t = 4.00$ s, we get $y = -18.4$ m and $v = -24.2$ m/s. Here, the ball has passed the highest point and is 18.4 m below the origin ($y$ is negative) and is moving downward ($v$ is negative) with a speed of 24.2 m/s.

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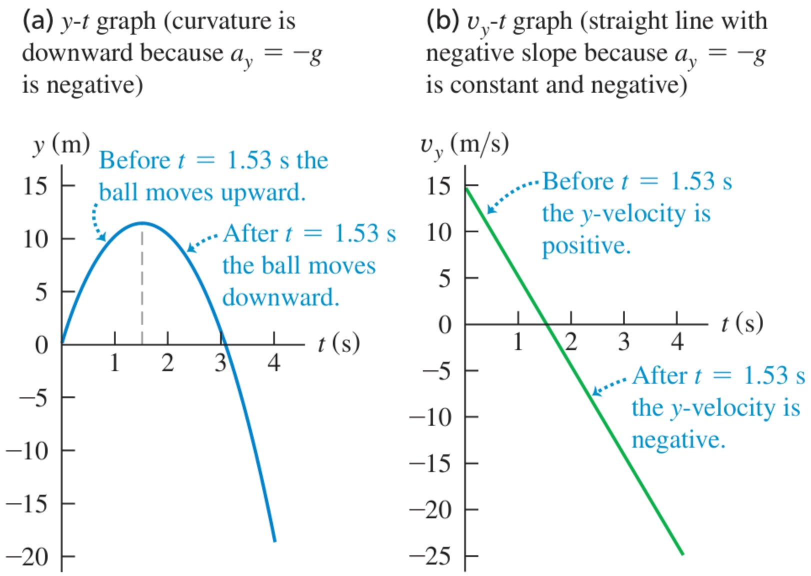


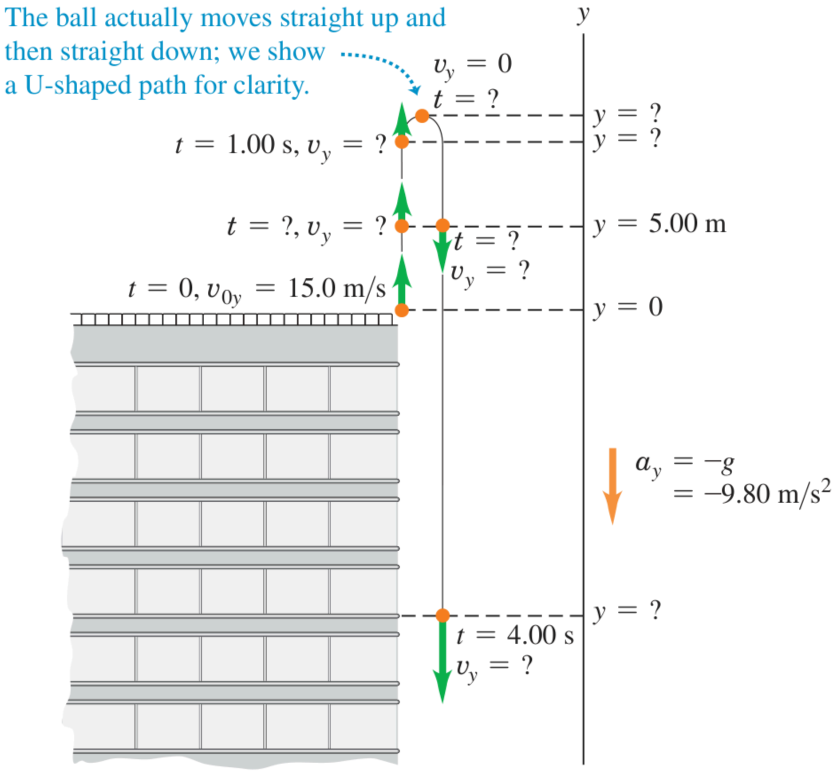

Questions? 🤔

# Problem solving 🧮

# Basic strategy

1. Identify 🔎
2. Set up 🗺️
3. Execute 🔧
4. Evaluate 👀

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   - In this course, you'll almost never encounter a situation in which acceleration isn't constant

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     - Some will be known, some unknown, some missing
     - Decide which unknowns are the target variables

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   - Translate the prose into physics
     - eg. “Where is he when his speed is 25 m/s?” means “What is $x$ when $v = 25$ m/s?”
     - Be alert for implicit info eg. “A car sits at a stop light” usually means $v_0 = 0$

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     - (more on this in the next section of the slides...)
   - Make any qualitative and quantiative predictions you can about the solution

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   - Are they within the general range of values that you expected?

# Questions?

*Checkpoint.* What is the basic strategy for problem solving again? 👀

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*Checkpoint.* What is the basic strategy for problem solving again? 👀

Do you see it? There's an ancronym: Oh, I see... stands for identify, set up, exec, eval

# Identify and set up the up-down example

*Example.* You throw a ball vertically upward from roof of a building. The ball leaves your hand with an upward speed of 15.0 m/s and is then in free fall. On its way back down, it just misses the railing. Find ball's position and velocity 1.00 s and 4.00 s after leaving your hand.

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- Identify. The phrase "in free fall" means that acceleration is due to gravity, which is constant
  - This uses the concept of motion with constant acceleration which allows us to use the kinematic equations

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  - We note that the ball's velocity is zero when it is at its highest point

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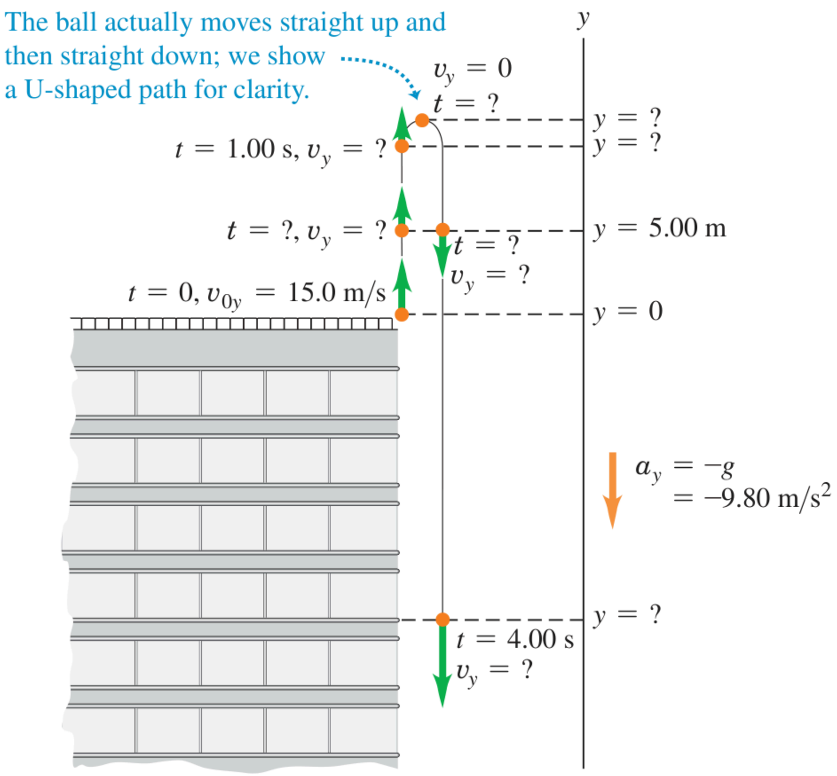
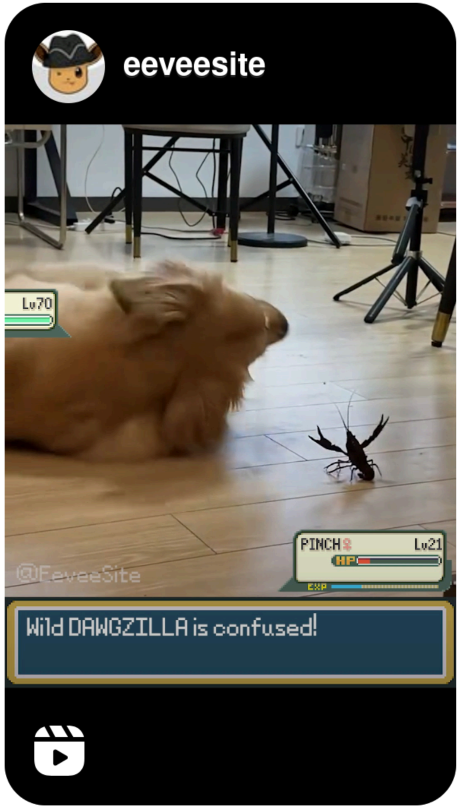


Questions? 😭

Brain break! 🧠 💤

# Poke-pets




Watch some `@eeveesite` reels 🐕

- `instagram.com/eeveesite`

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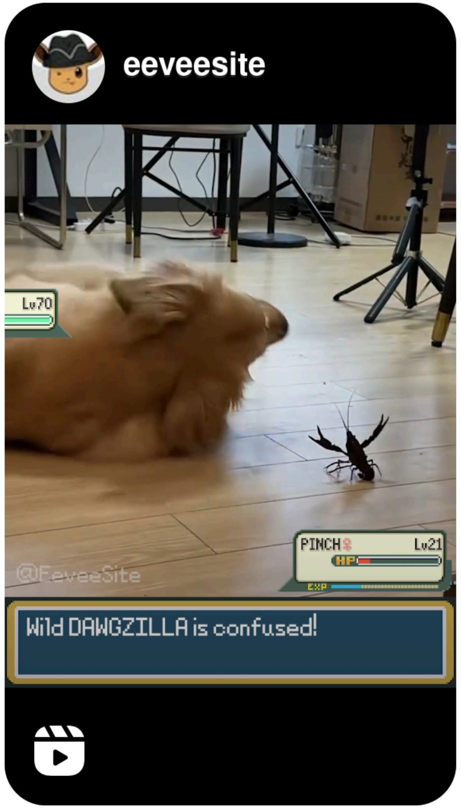


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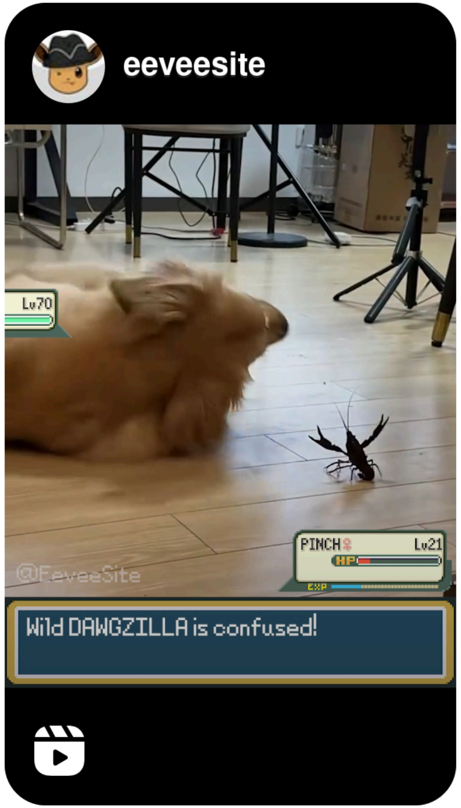


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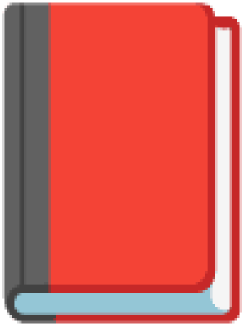
Nah, just kidding. (But also real, pls watch the reels. Reel and relax.) 😌

# Book vs paper

Drop a sheet of paper and a book, side by side. Which falls faster?

Now, place the paper below the book then drop. What happens?

Finally, place the paper on top of the book then drop them. What happens?



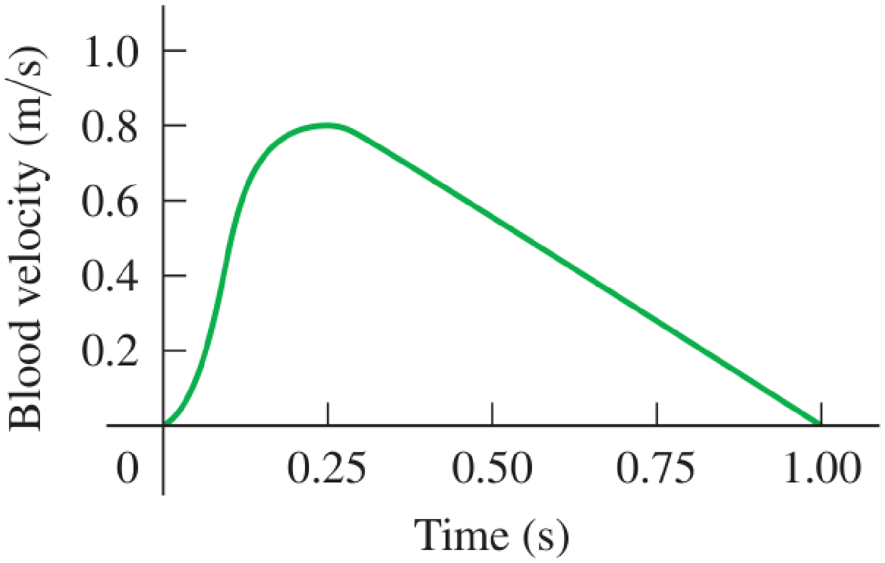
# Book vs paper

It's actually quite simple. The book "plows through the air" leaving an air resistance-free path for the paper to follow!

# Graphical analysis 📈

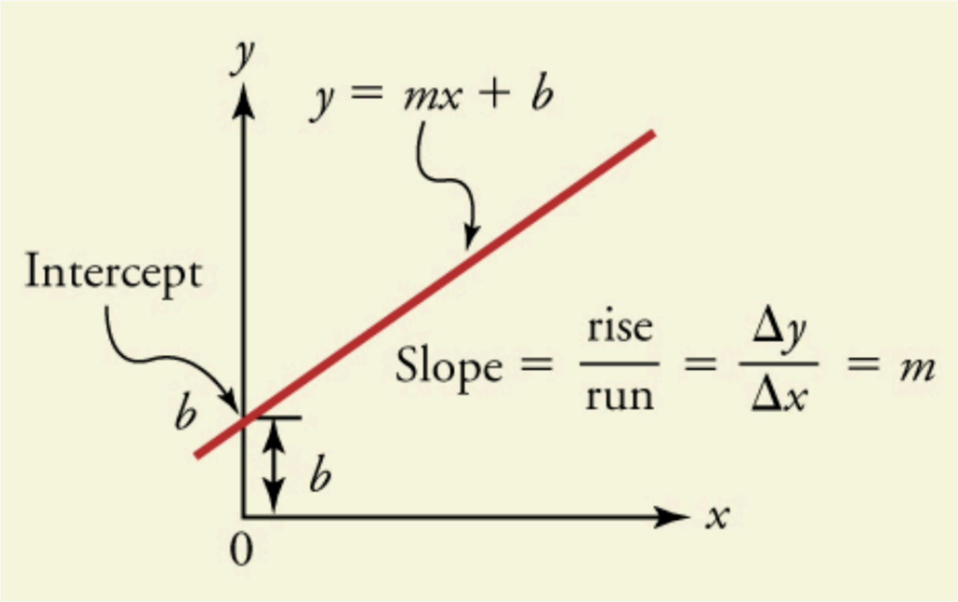
# Slopes and general relationships

- A graph, like a picture, is worth a thousand words 🧀
- Not only they contain numerical info, they also show relationships between physical quantities



# Slopes and general relationships

- Graphs have perpendicular axes: horizontal and vertical
- Horizontal axis is usually an **independent variable**, and vertical a **dependent var.**
  - If we call hor axis the $x$-axis and ver the $y$-axis, a straight-line graph has general form $y = mx + b$



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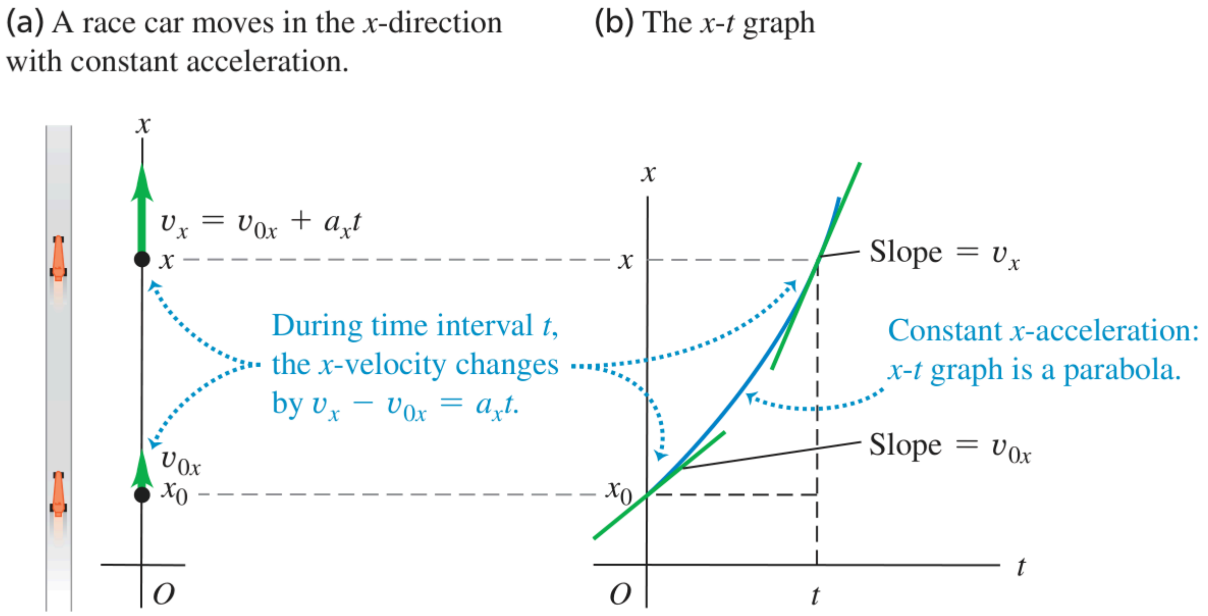

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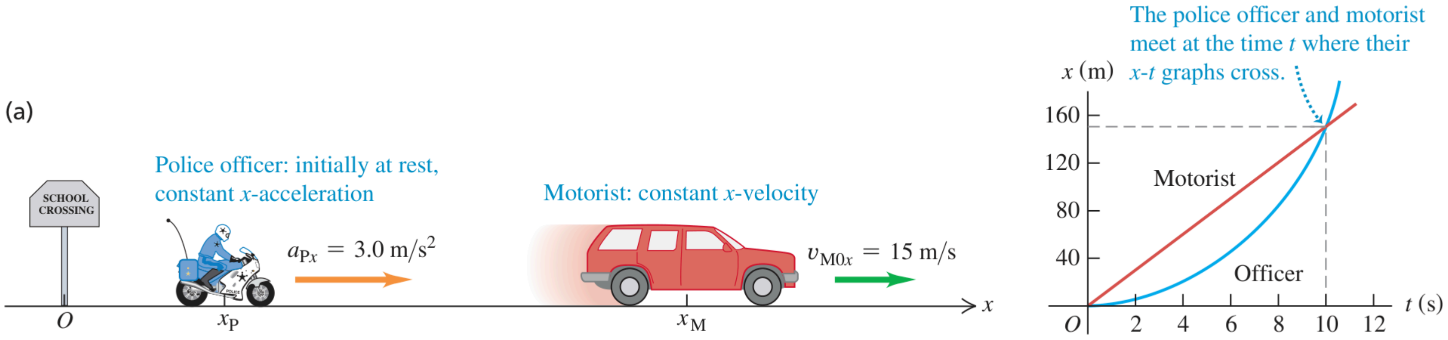

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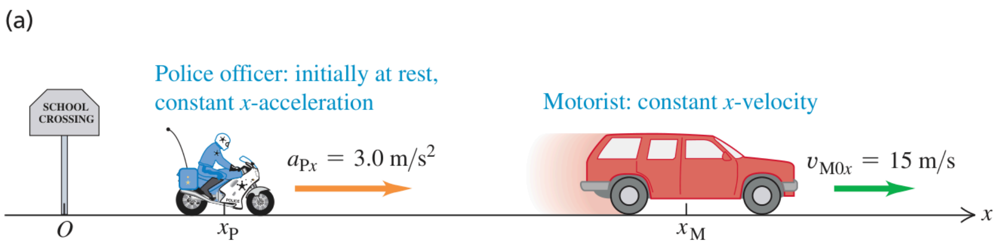


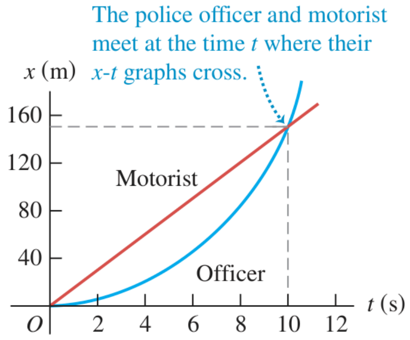


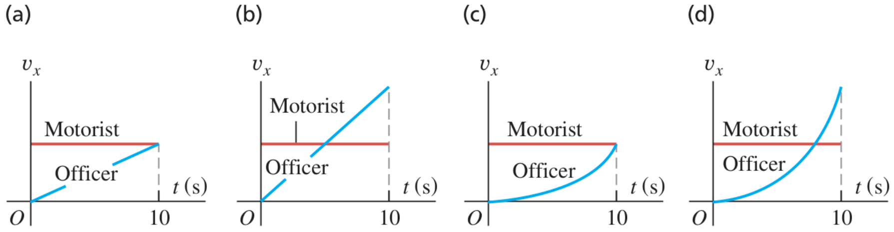


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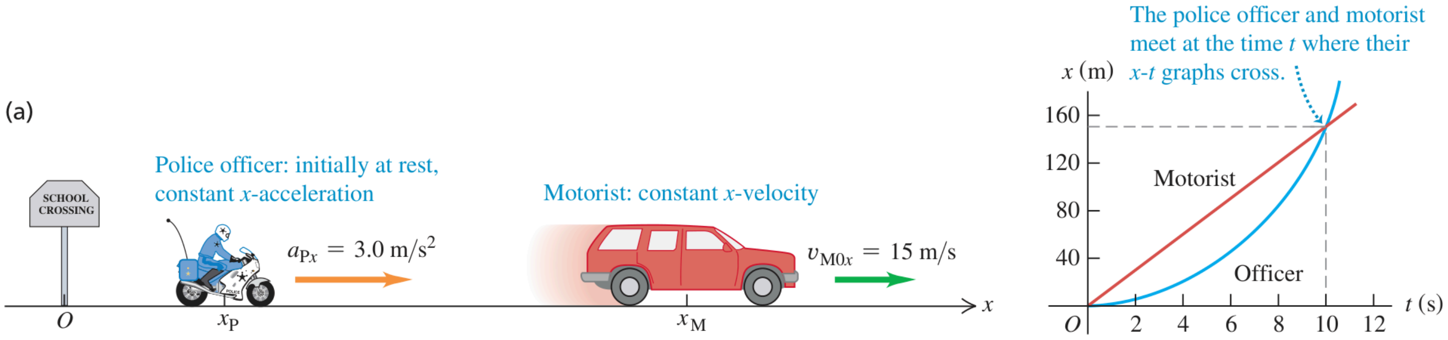
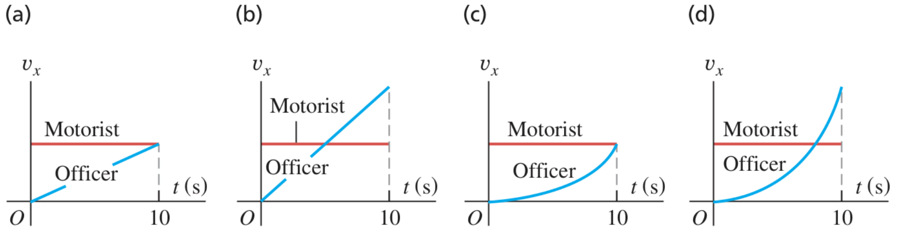


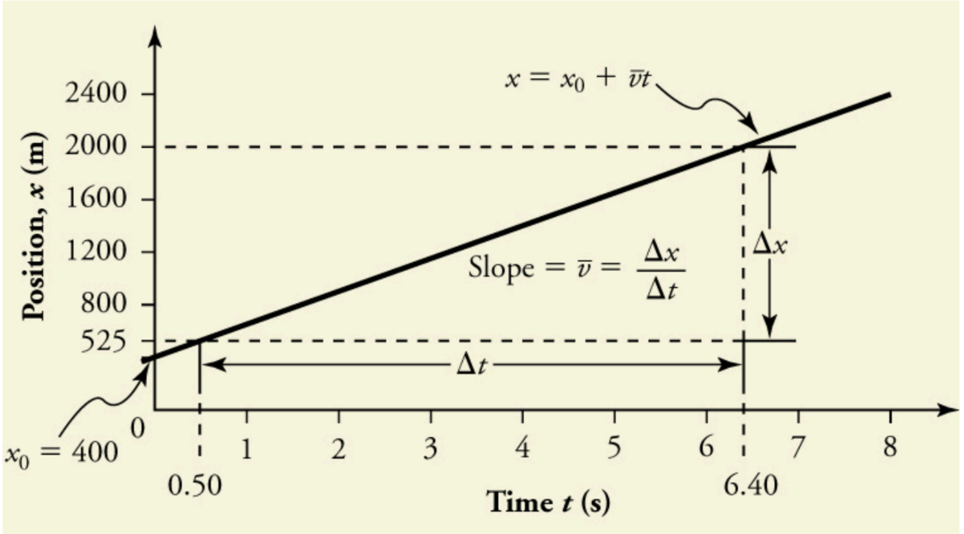




What do you think is the corresponding $v$-$t$ graph?

- It's B

# Position vs time graph (for $a = 0$, $v$ is constant)

- Time is usually an independent variable that other quantities depend upon
- Observe that $x = x_0 + \overline{v}t$ is analogous to $y = mx + b$
- Here, slope is average velocity $\overline{v}$



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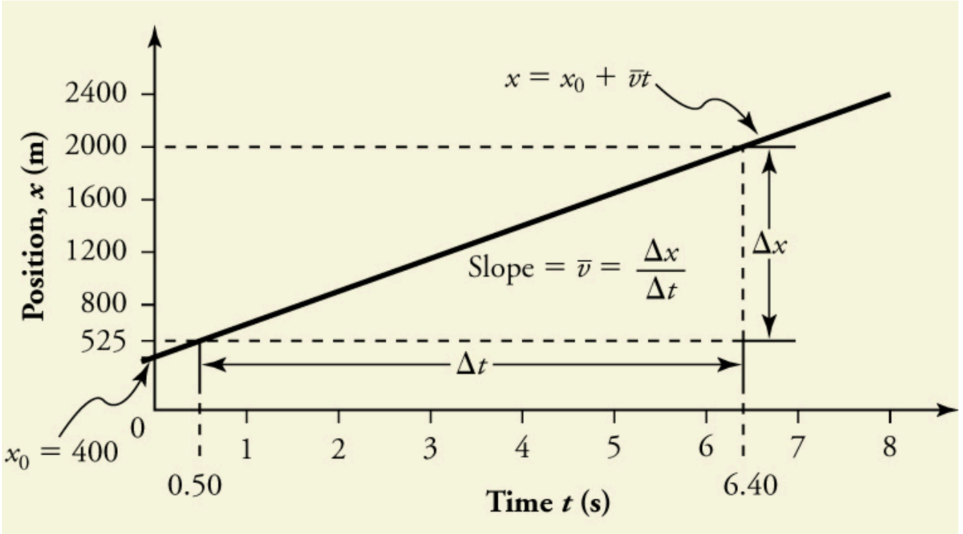


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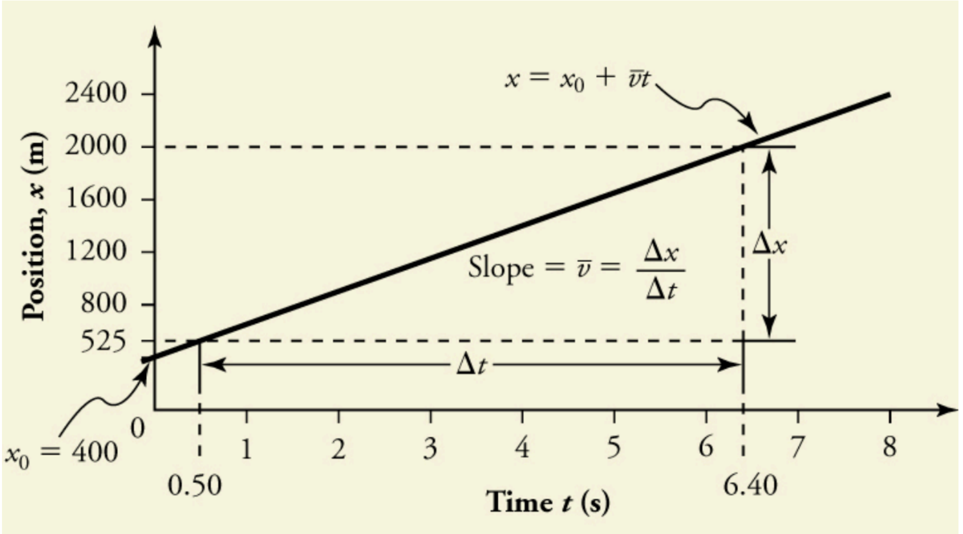


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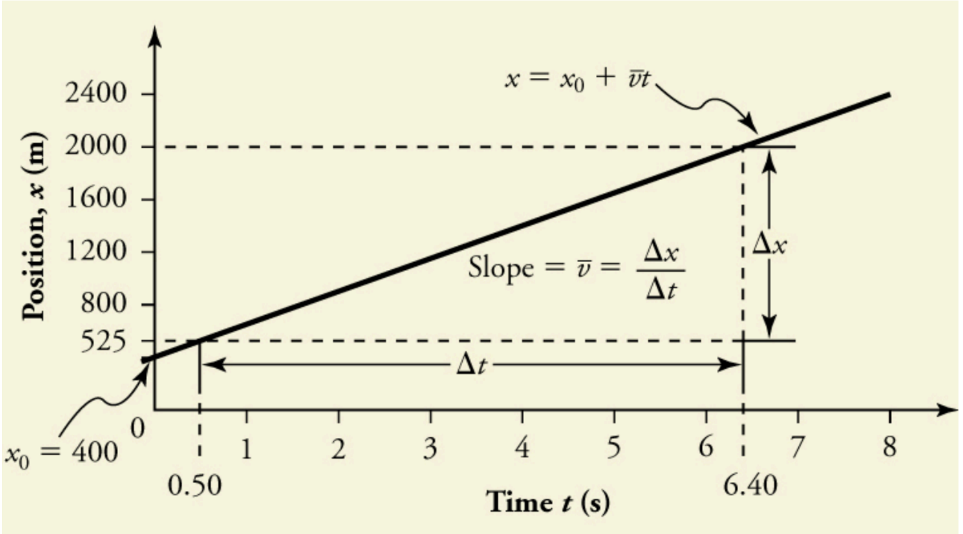


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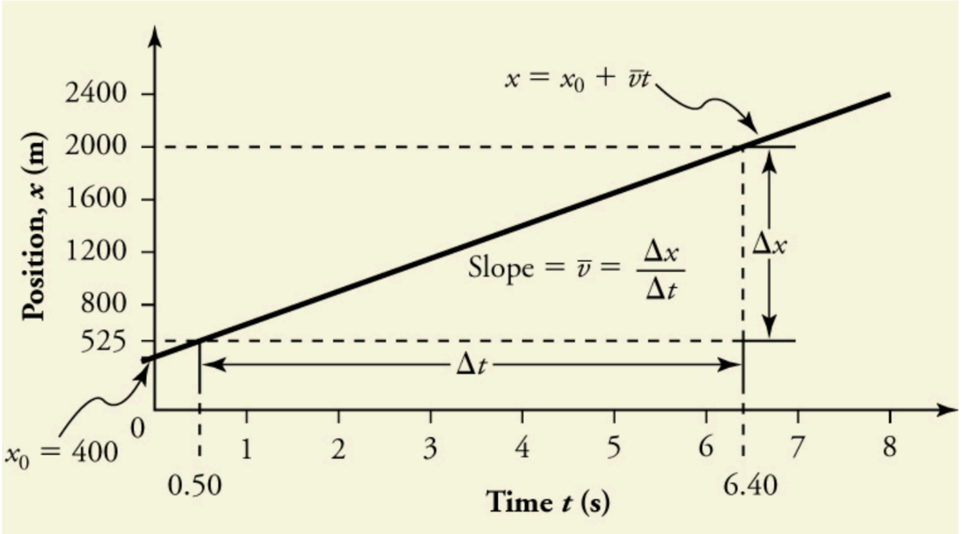


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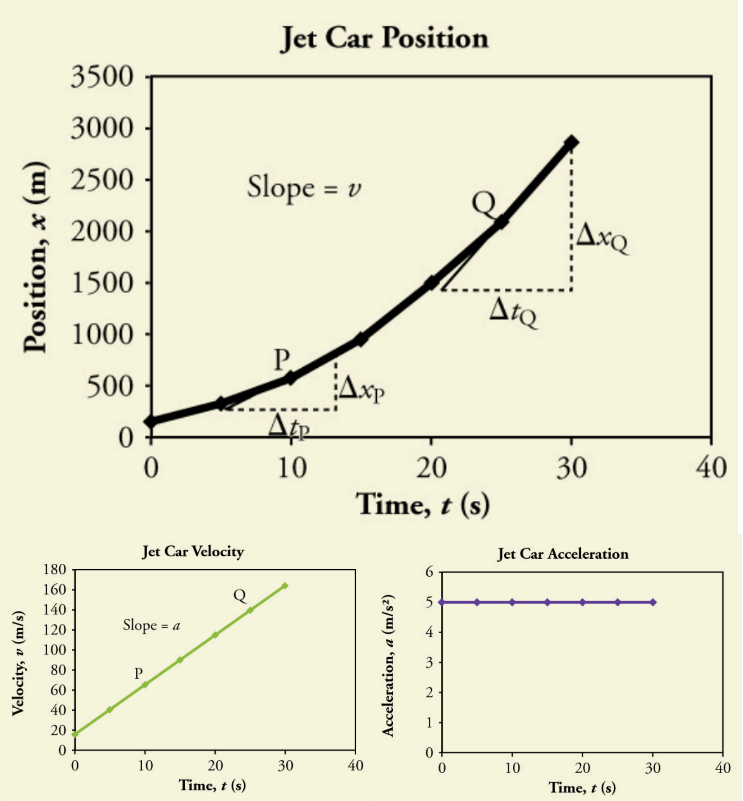
- This is large, much greater than typical road speed limit

# Graphs of motion (for constant $a$ but $a \neq 0$)

- The $x$-$t$ graph is a curve rather than a straight line
- Slope becomes steeper as time evolves, showing $v$ increases over time
- Slope at any point on $x$-$t$ is instantaneous $v$ at that point
- Slope of $v$-$t$ is acceleration



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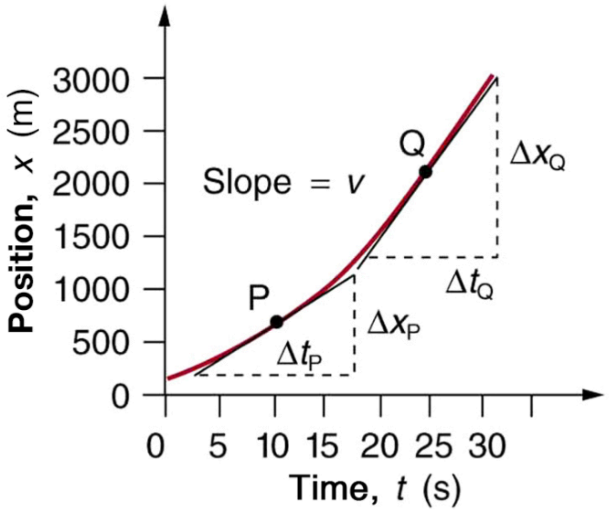


| $t$ (s) | $x$ (m) |
|---|---|
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| 10 | 600 |
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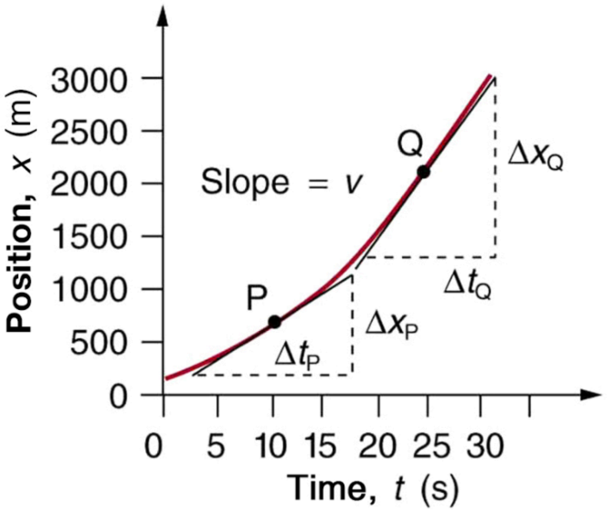


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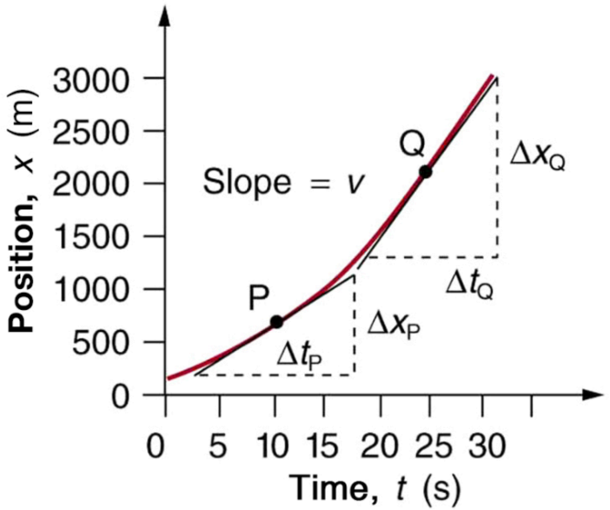
- Find tangent line at $t = 25$ s
- 

# Graphs of motion (for constant $a$ but $a \neq 0$)




| $t$ (s) | $x$ (m) |
|---|---|
| 0 | 200 |
| 5 | 338 |
| 10 | 600 |
| 15 | 988 |
| 20 | 1500 |
| 25 | 2138 |
| 30 | 2900 |

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- Pick 2 endpoints of tangent: (19 s, 1300 m),(32 s, 3120 m)
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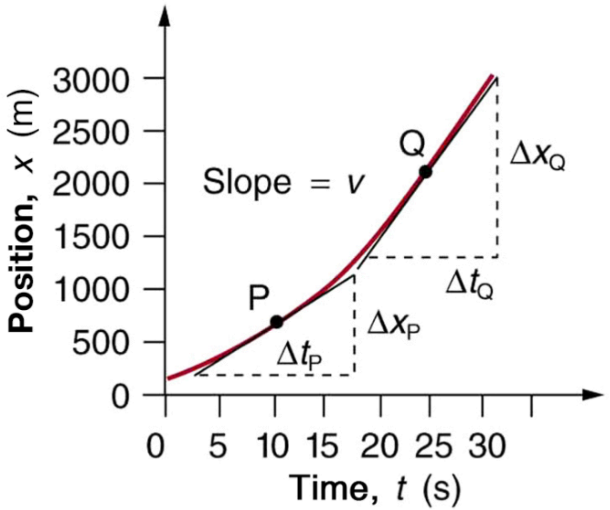


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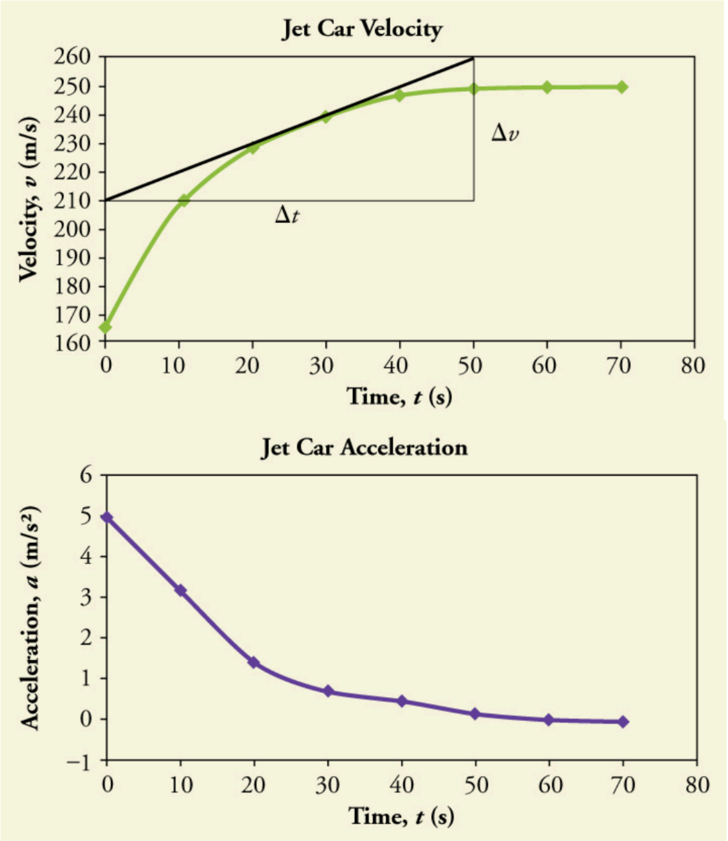
- $v = \frac{\Delta x}{\Delta t} = \frac{(3120-1300)\text{m}}{(32-19)\ \text{s}} = 140\frac{\text{m}}{\text{s}}$

# Graphs of motion (for non-constant $a$)

- Same ideas as previous slide (constant $a \neq 0$)
- Slope of $v$-$t$ is acceleration
- But $v$-$t$ graph is a curve rather than a straight line
- Slope becoming steeper as time evolves shows $a$ increasing over time



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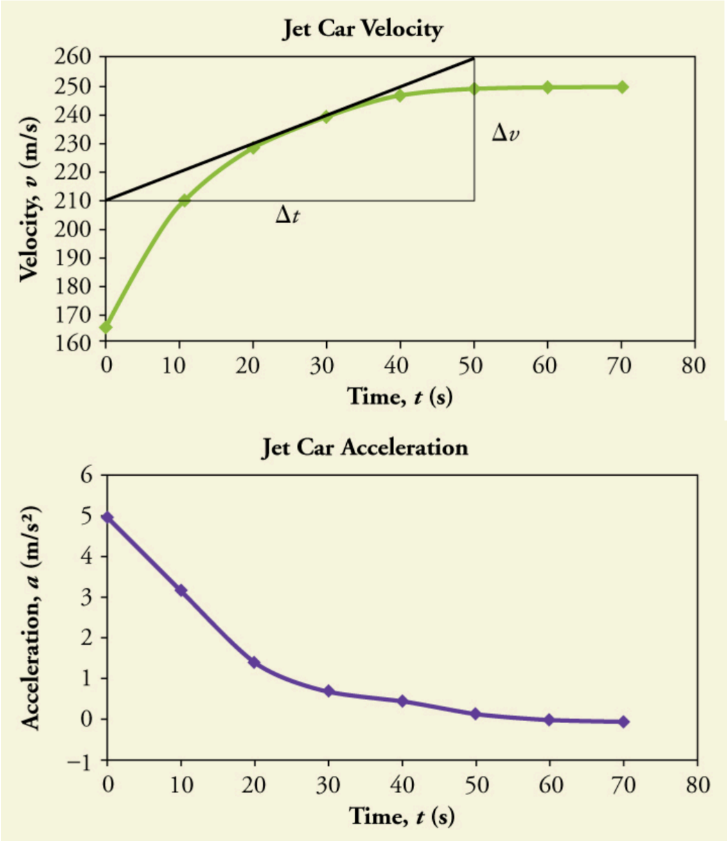


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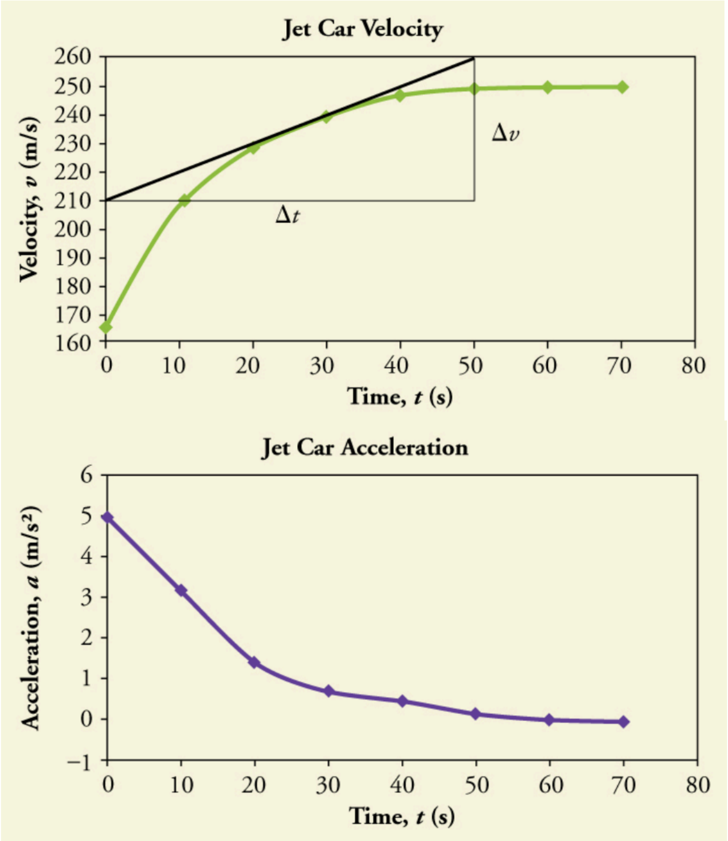


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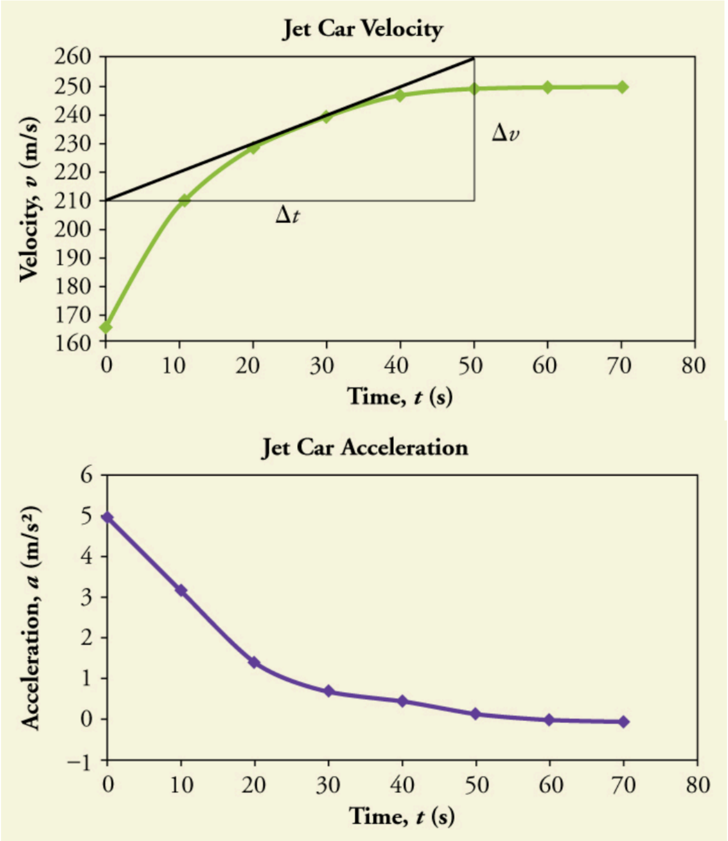


*Example.* Find the accel. of jet at $t = 25$ s by finding slope.

- Find tangent line at $t = 25$ s
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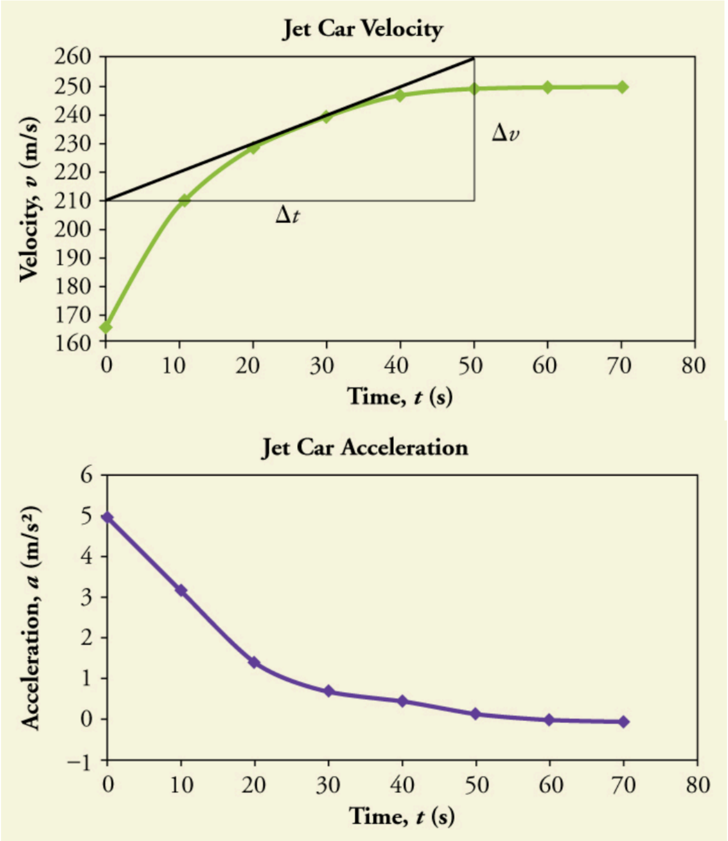


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- Find tangent line at $t = 25$ s
- Pick 2 endpoints of tangent: $(1.0 \text{ s}, 210 \frac{\text{m}}{\text{s}})$, $(51 \text{ s}, 260 \frac{\text{m}}{\text{s}})$
- $a = \frac{\Delta v}{\Delta t} = \frac{(260-210) \text{ m/s}}{(51-1.0) \text{ s}} = 1.0 \frac{\text{m}}{\text{s}^2}$

Questions? 😴

*Checkpoint.* If the kinematic equation $v = v_0 + at$ is analogous to the equation of line $y = mx + b$, what are the corresponding variables? 📉

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$v$ is dep var $y$, $v_0$ is intercept $b$,
$a$ is slope $m$, $t$ is indep var $x$

## Some notes 📎

You can play around with $x$-$t$, $v$-$t$ and $a$-$t$ on PhET (click here)

# Quiz time 🕐

# Sailing through graphs 🌊 🚢

The graph of velocity vs. time of a ship coming into a harbor is shown below. (a) Describe the motion of the ship based on the graph. (b) What would a graph of the ship's acceleration look like?