One-dimensional kinematics, continued

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2025 W34[1]

[1]Phys 20.01 Mod 1. All figures are from Urone (2022), Hewitt (2024), Young and Freedman (2019) unless noted.

Agenda

Previously ⏪

Motion with constant acceleration ⭕

Freely falling objects 🪶

Quiz time 🕐

Previously ⏪

All about motion, displacement, velocity, acceleration

Who's faster?

Example. Which undergoes greater acceleration: an airplane that goes from 1000 km/h to 1005 km/h in 10 seconds or a skateboard that goes from zero to 5 km/h in 1 second?

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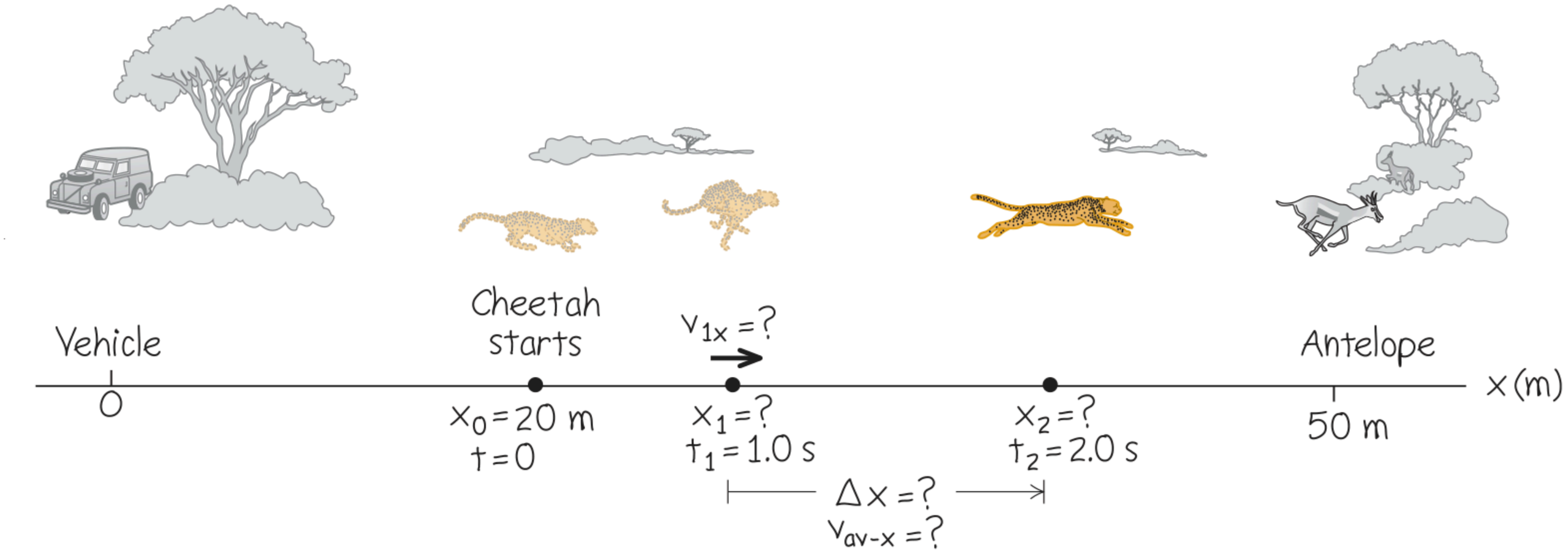
Oh dear, antelope!

Example. A cheetah is crouched 20 m to the east of a vehicle. At time $t = 0$, the cheetah begins to run due east toward an antelope that is 50 m to the east of the vehicle. During the first 2.0 s of the chase, the cheetah's x-coordinate varies with time according to

$$x = 20\ \mathrm{m} + (5.0\ \mathrm{m/s^2})t^2.$$

(a) Find cheetah's displacement between $t_1 = 1.0$ s and $t_2 = 2.0$ s.
(b) Find its average velocity during that interval.



1 Point axis in direction cheetah runs, so that all values will be positive.

2 Place origin at vehicle.

3 Mark initial positions of cheetah and antelope.

4 Mark positions for cheetah at 1 s and 2 s.

5 Add the known and unknown quantities.

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- $x_2 = 20 \text{ m} + (5.0 \text{ m/s}^2)\left(\underbrace{2.0} \text{ s}\right)^2 = 40 \text{ m}.$

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Displacement during this interval $\Delta t = \left(\underbrace{2.0} - \underline{1.0}\right)$ s $= 1.0$ s is

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(b) The average velocity during this interval is

- $$\bar{v} = \frac{\Delta v}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(40-20)\ \mathrm{m}}{(2.0-1.0)\ \mathrm{s}} = \frac{15\ \mathrm{m}}{1.0\ \mathrm{s}} = 15\ \mathrm{m/s}.$$

Questions? 🤔

Motion with constant acceleration ⭕

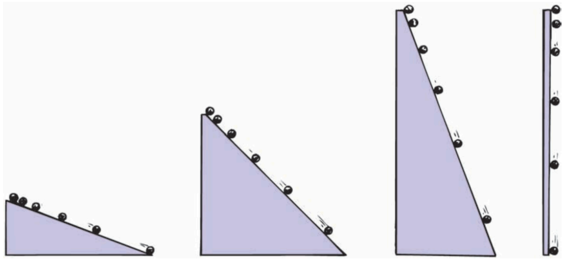
Predicting motion using equations

- We might know that the greater the acceleration a, eg. 🚗 💨 ... 🚸 , the greater the displacement Δx given time
  - But, we don't have specific equation relating a and Δx
  - Now, we develop some convenient equations for kinematic relationships

Constant acceleration

- The simplest kind of accelerated motion is straight-line motion with constant acceleration
  - Here, velocity changes at the same rate throughout motion

Example. Falling 🏀 if effects of air aren't important, 📦 sliding on an incline, ✈️ being catapulted from deck of an aircraft carrier



Kinematic equations

- When acceleration is constant, these **kinematic equations** relate the position x and velocity v at any time t to initial position x_0, initial velocity v_0 (both measured at time $t_0 = 0$), and acceleration a

$$x = x_0 + \overline{v}t, \qquad v = v_0 + at, \qquad \overline{v} = \frac{1}{2}(v_0 + v),$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2, \qquad v^2 = v_0^2 + 2a(x - x_0).$$

- In vertical motion, y takes the place of x

Deriving kinematic equations

- Constant acceleration means $\overline{a} = a =$ some constant value
- Set initial time $t_0 = 0$ so that $\Delta t = t - 0 = t$
- For the first,

$$\begin{aligned}
\overline{v} &= \Delta x / \Delta t && \text{define average velocity} \\
\Rightarrow \overline{v}\Delta t &= \Delta x && \text{multiply both sides by } \Delta t \\
\Rightarrow \overline{v}t &= x - x_0 && \text{expand } \Delta x \text{ and } \Delta t \\
\Rightarrow \overline{v}t + x_0 &= x && \text{add } x_0 \text{ to both sides}
\end{aligned}$$

- Similarly, for the second,

$$\begin{aligned}
\overline{a} = a = \Delta v/\Delta t \qquad & \text{def average acceleration} \\
\Rightarrow a\Delta t = \Delta v \qquad & \text{multiply } \Delta t \\
\Rightarrow at = v - v_0 \qquad & \text{expand } \Delta v, \Delta t \\
\Rightarrow at + v_0 = v \qquad & \text{add } v_0
\end{aligned}$$

- For the third, we note that when a is constant, $\overline{v}$ is just simple average of velocities at beginning and end of the time interval

$$\overline{v} = \frac{1}{2}(v_0 + v)$$

- For the fourth, we combine first, second and third

$$\begin{aligned}
\overline{v} &= (v_0 + v)/2 && \text{recall third}\\
\Rightarrow \overline{v} &= (v_0 + [v_0 + at])/2 && \text{substitute second into } v\\
\Rightarrow \overline{v} &= v_0 + at/2 && \text{simplify}
\end{aligned}$$

$$\begin{aligned}
x &= x_0 + \overline{v}t && \text{recall first}\\
\Rightarrow x &= x_0 + \left[v_0 + \frac{1}{2}at\right]t && \text{sub above} \to \overline{v}\\
\Rightarrow x &= x_0 + v_0 + \frac{1}{2}at^2 && \text{simplify}
\end{aligned}$$

- For the fifth, we also combine first, second and third

$$
\begin{aligned}
v &= v_0 + at && \text{recall second} \\
\Rightarrow (v - v_0)/a &= t && \text{subtract } v_0\text{, divide } a \\
x &= x_0 + \bar{v}t && \text{recall first} \\
\Rightarrow x &= x_0 + \left[\frac{1}{2}(v_0 + v)\right]\left[\frac{v - v_0}{a}\right] && \text{sub above} \rightarrow t, \text{third} \rightarrow \bar{v} \\
\Rightarrow x &= x_0 + \frac{1}{2a}(v^2 - v_0^2) && \text{multiply altogether} \\
\Rightarrow 2a(x - x_0) + v_0^2 &= v^2 && \text{subtract } x_0, \text{multiply } 2a, \text{add } v_0^2
\end{aligned}
$$

Questions? 🙄

Checkpoint. Which kinematic equation should we use if we are looking for the value of initial position? 🧮

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3 possible equations to use to look for x_0:

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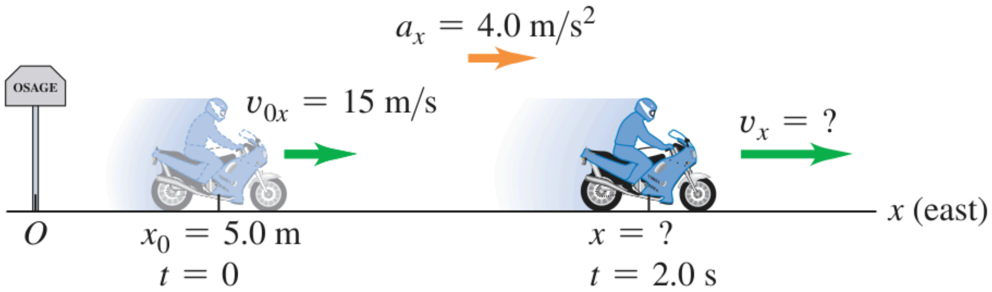
3 possible equations to use to look for x_0:

$$x = x_0 + \overline{v}t, \qquad x = x_0 + v_0 t + \frac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Just motorcyclin'

Example. A motorcyclist heading east through a small town accelerates at constant 4.0 m/s^2 after he leaves the city limits. At time $t = 0$, he is 5.0 m east of city-limits signpost while he moves east at 15 m/s. (a) Find his position and velocity at $t = 2.0$ s. (b) Where is he when his speed is 25 m/s?



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(a) Since we know the values of x_0, v_0 and a, we can use the fourth equation for x and the second for v, given time $t = 2.0$ s:

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$$\begin{aligned} x &= x_0 + v_0 t + \frac{1}{2} a t^2 \\ &= 5.0\ \mathrm{m} + (15\ \mathrm{m/s})(2.0\ \mathrm{s}) + \frac{1}{2}(4.0 \mathrm{m/s^2})(2.0\ \mathrm{s})^2 \\ &= 5.0\ \mathrm{m} + \left(15 \frac{\mathrm{m}}{\cancel{\mathrm{s}}}\right)(2.0\ \cancel{\mathrm{s}}) + \frac{1}{2}\left(4.0 \frac{\mathrm{m}}{\cancel{\mathrm{s}^2}}\right)(4.0\ \cancel{\mathrm{s}^2}) \\ &= 5.0\ \mathrm{m} + 30\ \mathrm{m} + 8\ \mathrm{m} \qquad = 43\ \mathrm{m} \end{aligned}$$

Just motorcyclin'

$$\begin{aligned} v &= v_0 + at \\ &= 15\,\frac{\text{m}}{\text{s}} + \left(4.0\,\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(2.0\,\cancel{\text{s}}) \\ &= 15\,\frac{\text{m}}{\text{s}} + 8\,\frac{\text{m}}{\text{s}} \\ &= 23\,\frac{\text{m}}{\text{s}} \end{aligned}$$

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$$v^2 = v_0^2 + 2a(x - x_0).$$

To isolate x, we subtract both sides by v_0^2, divide by $2a$, add x_0:

$$\Rightarrow v^2 - v_0^2 = 2a(x - x_0)$$

$$\Rightarrow (v^2 - v_0^2)/2a = x - x_0$$

$$\Rightarrow (v^2 - v_0^2)/2a + x_0 = x$$

Just motorcyclin'

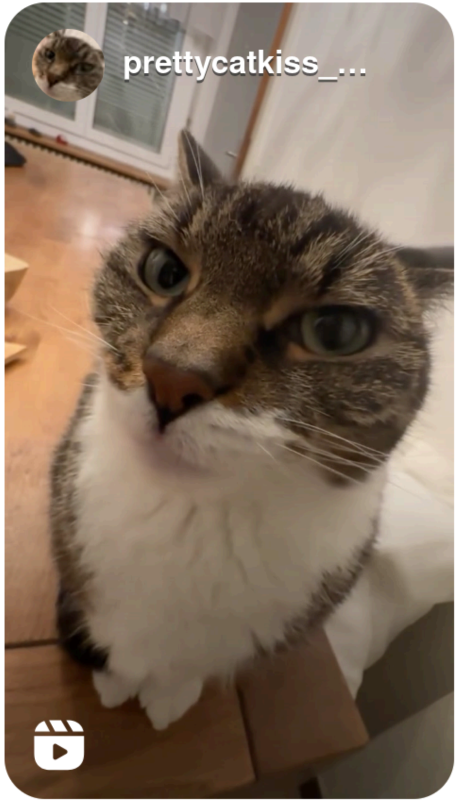
Substituting the known values,

$$\begin{aligned} x &= \frac{v^2 - v_0^2}{2a} + x_0 \\ &= \frac{(25\ \mathrm{m/s})^2 - (15\ \mathrm{m/s})^2}{2(4.0\ \mathrm{m/s^2})} + 5.0\ \mathrm{m} \\ &= \frac{(25^2 - 15^2)\frac{\mathrm{m}^{\cancel{2}}}{\cancel{\mathrm{s}^2}}}{8\frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}^2}}} + 5.0\ \mathrm{m} \\ &= 50\ \mathrm{m} + 5.0\ \mathrm{m} \qquad = 55\ \mathrm{m} \end{aligned}$$

Questions? 😭

Brain break! 🧠 💤

Cat defense



Watch these cat reels 🐱

- `instagram.com/p/DM-_0l3oTLk`
- `instagram.com/p/DNW2AWqs3rX`
- `instagram.com/p/DH0MJRHubQ4`

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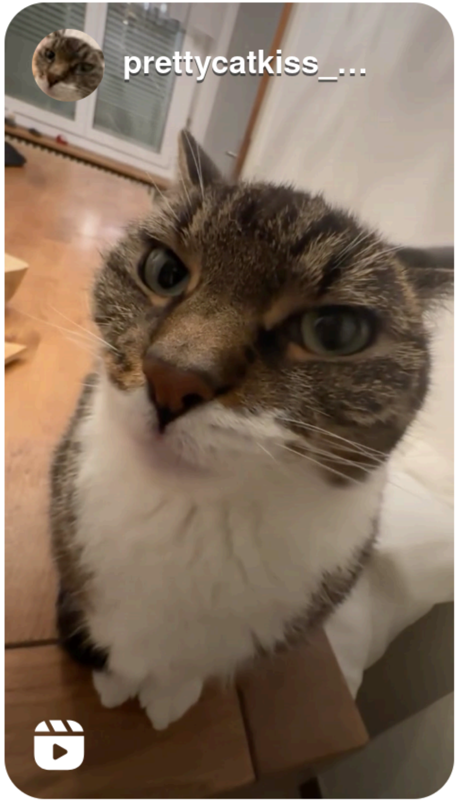

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What is the cat being subjected to in the third reel?

-

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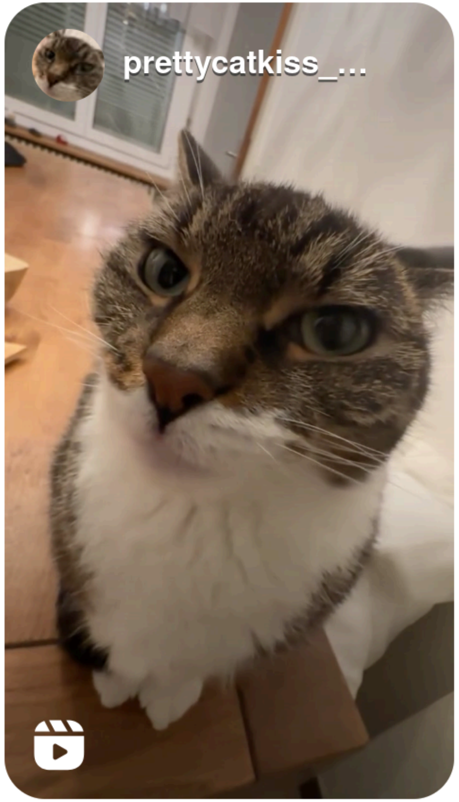

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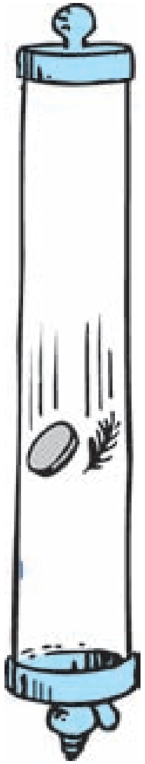
What is the cat being subjected to in the third reel?

- A futile attack from a bunny moving at constant speed but not constant velocity 😵

Freely falling objects 🪶

Free fall

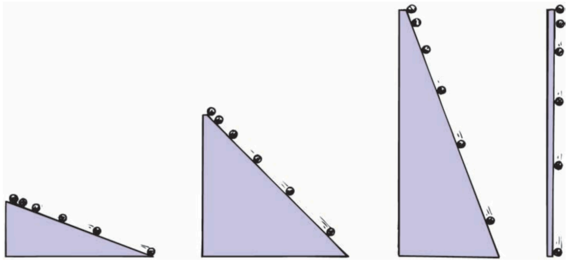
- An object in **free-fall** moves with constant acceleration if air resistance is negligible
  - Here only gravity affects the motion
- On earth, free falling objects have an acceleration g due to gravity at $g = 9.8\ \mathrm{m/s^2} \approx 10\ \mathrm{m/s^2}$

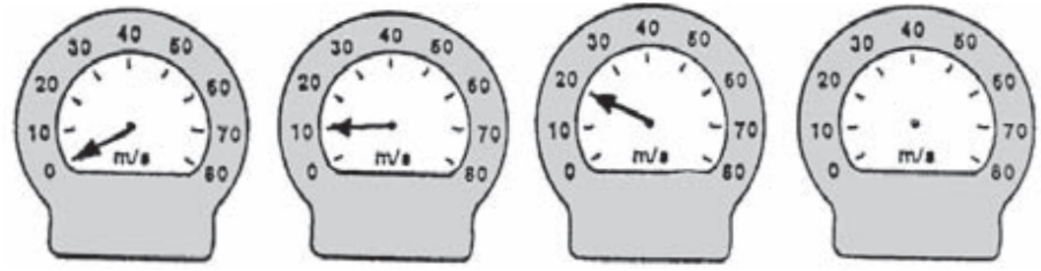


Free fall

- Acceleration a_y along the vertical can be taken either as $+a_y$ or $-a_y$ depending on your choice of coordinate system
  - If upward is positive, $a_y = -g$ is negative, otherwise $a_y = g$. Former is the typical choice
- Since a is constant in free fall, you can use previously discussed kinematic equations where either $a = \pm g$ and y takes place of x

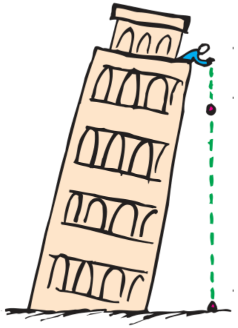
How fast

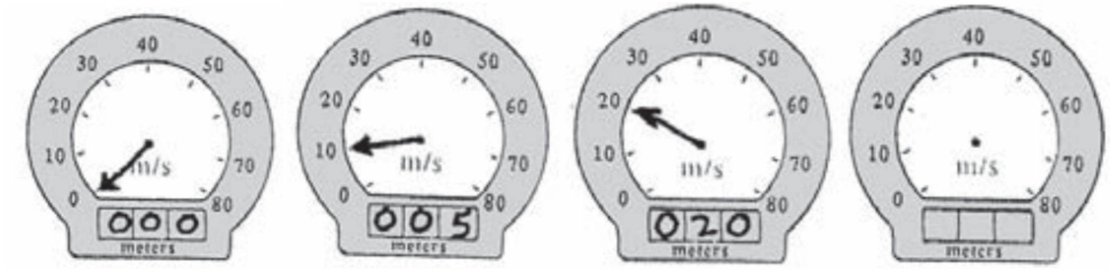




- Galileo found that a ball rolling down an inclined plane picks up the same amount of speed in successive seconds
  - During each second of fall, the object gains a speed of 10 m/s
  - As in, it gains 10 m/s/s = 10 m/s^2 (in fact, it's acceleration g)
  - Using one of the kinematic equations, we can compute it: speed $s = gt$ (from $v = v_0 + at$ where $v_0 = 0$)

How far





- How far an object falls is entirely different from how fast it falls
  - Galileo found that the distance d a uniformly accelerating object travels is proportional to the square of the time
  - Using one of the kinematic equations, we can compute it: $d = \frac{1}{2}gt^2$ (from $y = y_0 + v_0t + \frac{1}{2}at^2$ where $y_0 = v_0 = 0$)

A cat on the ledge

Example. A cat steps off a ledge and drops to the ground in 1/2 second. (a) What is its speed on striking the ground? (b) What is its average speed during the 1/2 second? (c) How high is the ledge from ground?


[1]

[1] vm.tiktok.com/ZP8BHu1DL

A cat on the ledge

- The speed is $s = gt = (10 \text{ m/s}^2)(1/2 \text{ s}) = 5 \text{ m/s}$
-

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- The speed is $s = gt = (10\ \mathrm{m/s^2})(1/2\ \mathrm{s}) = 5\ \mathrm{m/s}$
- Use kin. eq: $\overline{v} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(0 + 5)\ \mathrm{m/s} = 2.5\ \mathrm{m/s}$
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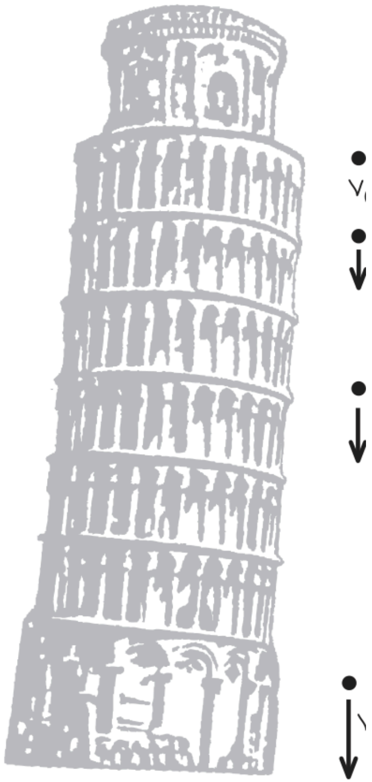
A freely falling coin

Example. A one-peso coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s? Ignore air resistance.



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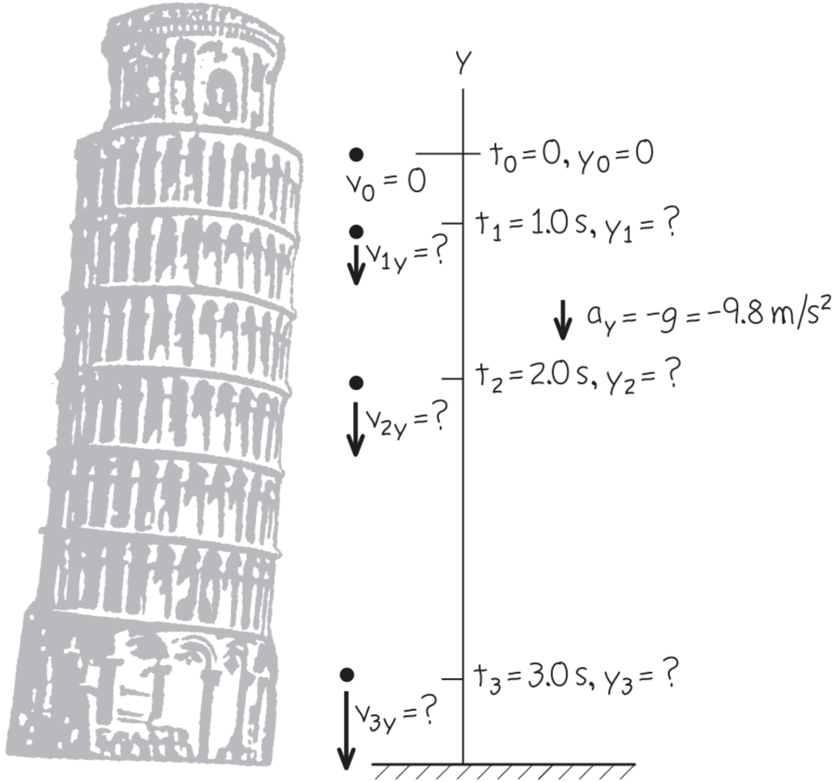

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$$y = y_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(-g)t^2 = (-4.9\ \mathrm{m/s^2})t^2,$$
$$v = v_0 + at = 0 + (-g)t = (-9.8\ \mathrm{m/s^2})t.$$

A freely falling coin

When $t = 1.0$ s,

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which means after 1.0 s, the coin is 4.9 m below origin (y is negative) and has a downward velocity (v is negative) with magnitude 9.8 m/s.

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Similarly, the results are $y = -20$ m and $v = -20$ m/s at $t = 2.0$ s, and $y = -44$ m and $v = -29$ m/s at $t = 3.0$ s.

Questions? 😴

Quiz time 🕐

Moving faster and faster and faster 🎱

Consider a billiards ball fired straight downward from a high-altitude balloon. If the muzzle velocity is 68.9 m/s and air resistance can be neglected, what is the acceleration of the ball after 3 seconds?